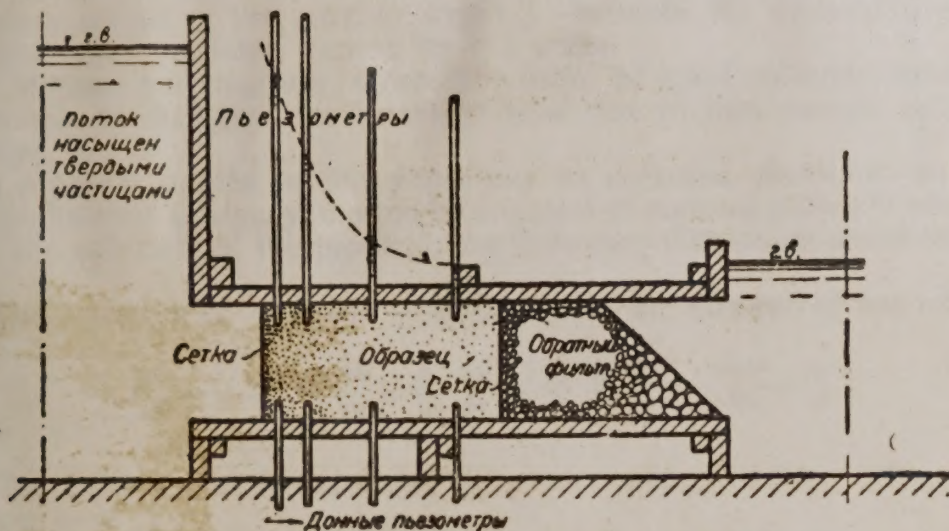


искусственная шероховатость. По оси этого щитка были выведены пьезометрические трубки. Кроме того в дне лотка были выведены контрольные пьезометрические трубки, которые также еще служили отверстиями для взятия проб на исследование концентрации потока.

Подача раствора в лоток производилась помощью насоса из специального бака, находящегося под лотком. Раствор, попадающий в бак с холостых сливов лотка, а также специально установленная турбинка производили практически удовлетворительное перемешивание и взбалтывание оседающих с течением времени твердых частиц, которыми насыщен поток.

Всего было проведено 30 опытов; концентрация потока принималась в опытах от 1 и до 0,25%; насыщение потока производилось глинистыми и песчаными частицами крупностью от 0,005 до 0,0075 см. Заилению подвергались пески крупностью от 0,3 до 0,05 см.

Размеры образца, подверженного заилению, были таковы: высота 10 см., ширина 20 см., длина от 20 до 30 см., напор устанавливался от 20 до 40 см.



Фиг. 13. Опытная установка по исследованию гидродинамических элементов потока, изменяющихся в процессе заилиния. — Продольный разрез установки по оси.

Обращаясь к результатам сопоставления теоретического решения с опытами, отметим следующее.

1. Выработанная модель движения грунтового потока в основном подтвердилась, — т. е. твердые частицы, несомые потоком, отлагаются именно в тех местах фильтрационной трубки, где сила инерции, развиваемая движущимся потоком, прижимает их к стенкам последней.

2. Полученные в настоящей работе теоретические зависимости также в основном подтвердились. Наилучшие совпадения дают функции $Q = Q(t)$, и $H = H(t)$, несколько худшие — $H = H(s)$. Расхождение с опытами в процентах колеблется в пределах от -13 до $+8\%$. И, наконец, надлежит отметить, что поток опытных точек на теоретические кривые ложится также в общем удовлетворительно.¹

Pressure Flow of Ground Water Carrying Fine Sandy and Clayey Particles.

By A. N. Patrashev, Eng.

The purpose of this paper is to show the theoretical solution of problems related to the pressure, stream-line (as far as the „basic“, rectilinear motion is concerned) motion of ground water carrying fine sand and clayey particles.

¹ См. подробнее в IV главе.

The size of solid particles carried by the stream is assumed to be such, that the velocity of particles has the magnitude and direction, differing only insignificantly from those of the water; this assumption makes it possible to regard such a stream (from a mechanical point of view) as a uniform system of material points.

In order to solve the problem, a method devised by Prof. N. N. Pavlovsky was used and a model of flow was conceived, in which the flow is represented by a system of curvilinear percolation tubes (see fig. 1); the cross-section of these tubes being variable with respect to time and length,—due to continuous deposition of solid particles carried by the water.

The centrifugal forces of inertia arising in the moving stream and due to the curved shape of percolation tubes,—constitute the basic cause of the deposition of solid particles; it is necessary to note that this deposition takes place in those parts of the path of motion, where the centrifugal forces are directed along the inner normal to the path, because in this case the frictional forces arise in the boundary region of the moving stream,—between the motionless particles of the soil and solid particles, carried by the water.

For streams carrying not more than 10% of solid particles, however, it is quite possible to disregard the losses of head due friction caused by the centrifugal forces.

The problem studied in this paper can be regarded, therefore, as a problem of two-dimensional unsteady motion of a system of material points of variable mass.

For the solution of the problem, the following differential equation have been derived:

1. The equation of dynamic equilibrium for an element of the tube

$$\frac{\rho}{\gamma} \frac{du}{dt} = -\frac{dz}{ds} + \frac{\partial h}{\partial s} - \frac{h}{\Omega} \frac{\partial \Omega}{\partial s} - \frac{\mu}{\gamma} \frac{\partial u}{\partial r} \frac{1}{D} - \frac{\tau \mu u^2}{\gamma R \Omega} \cdot \frac{np}{d}. \quad (1)$$

2. The equation of the change of specific energy, taking place along the percolation tube

$$\frac{\rho}{\gamma} \frac{du}{dt} = -\frac{dz}{ds} + \frac{\partial h}{\partial s} - h \frac{n \cdot p \cdot \omega_0}{d \Omega} - \frac{\mu}{\gamma} \frac{\partial u}{\partial r} \frac{1}{D} - \frac{\tau \mu u^2}{\gamma R \Omega} \cdot \frac{np}{d}, \quad (2)$$

where the term $h \cdot \frac{np \omega_0}{d \Omega}$ expresses the specific energy of solid particles which are getting detached from the stream of water.

3. The equation showing the change of discharge along the percolation tube.

Having in view that this change occurs only as a result of separation of a certain amount of solid particles, we have:

$$\frac{\partial q}{\partial s} + \frac{n \cdot p \cdot \omega_0}{d} \cdot u = 0. \quad (3)$$

4. The equation of continuity derived for the problem here considered by Saint-Venant; it can be written in the following form:

$$\frac{\partial q}{\partial s} + \frac{\partial \Omega}{\partial t} = 0. \quad (4)$$

5. The equation of lateral equilibrium, or, which is the same,—the equation of change of head at a given point of the stream.

The weight of solid particles detached from the stream, does not influence the conditions of lateral equilibrium of the moving stream; it is easy, therefore to obtain the following equation, which is, we think, very important not only for the problem here discussed, but also for the study of flow in open channels and pipes:

$$\frac{\partial H}{\partial t} + n \frac{\partial q}{\partial \Omega} = 0. \quad (5)$$

6. The equation of the deposition of solid particles. Considering the displacement of a volume of the stream, equal to $q \times 1$, during an element of time Δt , through a length ds , and taking into account the process of separation of solid particles, we have:

$$\frac{dq}{dt} = n \cdot \omega_0 \frac{\partial k}{\partial t} \cdot u. \quad (6)$$

7. The equation of the deposition of solid particles with respect to a radius of the percolation tube. From simple geometrical considerations, and having in view that solid particles are being deposited only in places where the centrifugal forces of inertia are directed along the inner normal,—we obtain:

$$p = n \cdot (r_{i0} - r_i). \quad (7)$$

8. The equation of change of the cross-section of the „solid discharge“:

$$\left. \begin{aligned} \frac{\partial \omega}{\partial t} + \omega_0 \frac{\partial p}{\partial t} &= 0 \\ \frac{\partial \omega}{\partial s} + \omega_0 \frac{\partial p}{\partial s} &= 0 \end{aligned} \right\} \quad (8)$$

9. The equation of the radius of curvature of the stream.
From simple geometrical considerations, we have:

$$R = R_0 \left[1 - 2a^2 + 2a^2 \sqrt{1 - a^2} \cdot \frac{1 - 2a^2 (1 - \sqrt{1 - a^2})}{1 - 2a^2 (1 + \sqrt{1 - a^2})} \right] \quad (9)$$

where $a = 4,2 \left(1 - \frac{r_i}{r_{i0}} \right)$.

10. The equations of the discharge of the stream:

$$\left. \begin{aligned} q(t) &= u(t) \cdot \Omega(t) \\ q(s) &= u(s) \cdot \Omega(s) \end{aligned} \right\} \quad (10)$$

Solving simultaneously the equations enumerated above and reducing them to a system of Jacobi's differential equations, which, as is well-known, can be integrated, we obtain the following:

1. The function $u = u(t)$, which can be found from equation

$$\frac{\frac{n_0^2}{2} de^{\frac{2}{2}}}{u_0 n_0^2} \cdot r_{i0} \left(\frac{n_0}{\sqrt{2 \ln \frac{u}{u_0} + n_0^2}} - 1 \right) = t - t_0; \quad (11)$$

2. The function $r_i = r_i(t)$,—from equation

$$\frac{\frac{n_0^2}{2} de^{\frac{2}{2}}}{u_0 n_0^2} \cdot r_{i0} \left(\frac{r_{i0}}{r_i} - 1 \right) = t - t_0; \quad (12)$$

3. The function $n = n(t)$,—from equation

$$\frac{\frac{u_0^2}{2} de^{\frac{2}{2}}}{u_0 n_0^2} \cdot r_{i0} \left(\frac{n_0}{n} - 1 \right) = t - t_0; \quad (13)$$

4. The function $Q = Q(t)$;

in cases where the deposits of solid particles in the voids of the soil are permeable,—the value of Q can be expressed by equation

$$Q = Q_0 \left\{ \frac{1}{(k+1)^2} + \frac{p_0'^2}{2} \left[\left(1 - \frac{1}{(k+1)^2} \right) - \frac{2}{r_0} \left(1 - \frac{1}{k+1} \right) \right] \right\}; \quad (14)$$

in cases where the permeability of deposits can be considered as negligible

$$Q = \frac{Q_0}{(k+1)^2} \quad (15)$$

where

$$k = \frac{(t-t_0) u_0 n_0^2}{2de^2 r_{10}}; \quad (16)$$

5. The function $H = H(t)$ is expressed by equation

$$H_0 - H = \frac{2dr_{10}}{n_0} \ln \left[\frac{n_0^2 u_0 (t-t_0)}{2dr_{10}} + 1 \right]; \quad (17)$$

6. The function $u = u(s)$, — from equation

$$\frac{2dr_{100}}{3n_{00}^2} \left[\left(\frac{u_{00}}{u} \right)^{\frac{3}{4}} - 1 \right] = s - s_0; \quad (18)$$

7. The function $\eta = \eta(s)$, — from equation

$$\frac{2dr_{100}}{3n_{00}^2} \left(\frac{\eta^3}{r_{100}^3} - 1 \right) = s - s_0; \quad (19)$$

8. The function $n = n(s)$, — from equation

$$\frac{2dr_{100}}{3n_{00}^2} \left(\frac{n_{00}^3}{n^3} - 1 \right) = s - s_0; \quad (20)$$

9. The function $Q = Q(s)$, in cases where the deposits in the pores of the soil are permeable, is expressed by

$$Q = \frac{Q_{00}}{(M+1)^{\frac{4}{3}}} \left\{ (M+1)^{\frac{2}{3}} + \frac{p_0'^2}{2} \left[\left(1 - (M+1)^{\frac{2}{3}} \cdot \frac{r_{100}^2}{r_{10}^2} \right) - \frac{2}{r_{10}} (1 - r_{100} (M+1)) \right] \right\}; \quad (21)$$

case where the permeability of the deposits is negligible

$$Q = \frac{Q_{00}}{(M+1)^{\frac{4}{3}}}; \quad (22)$$

where

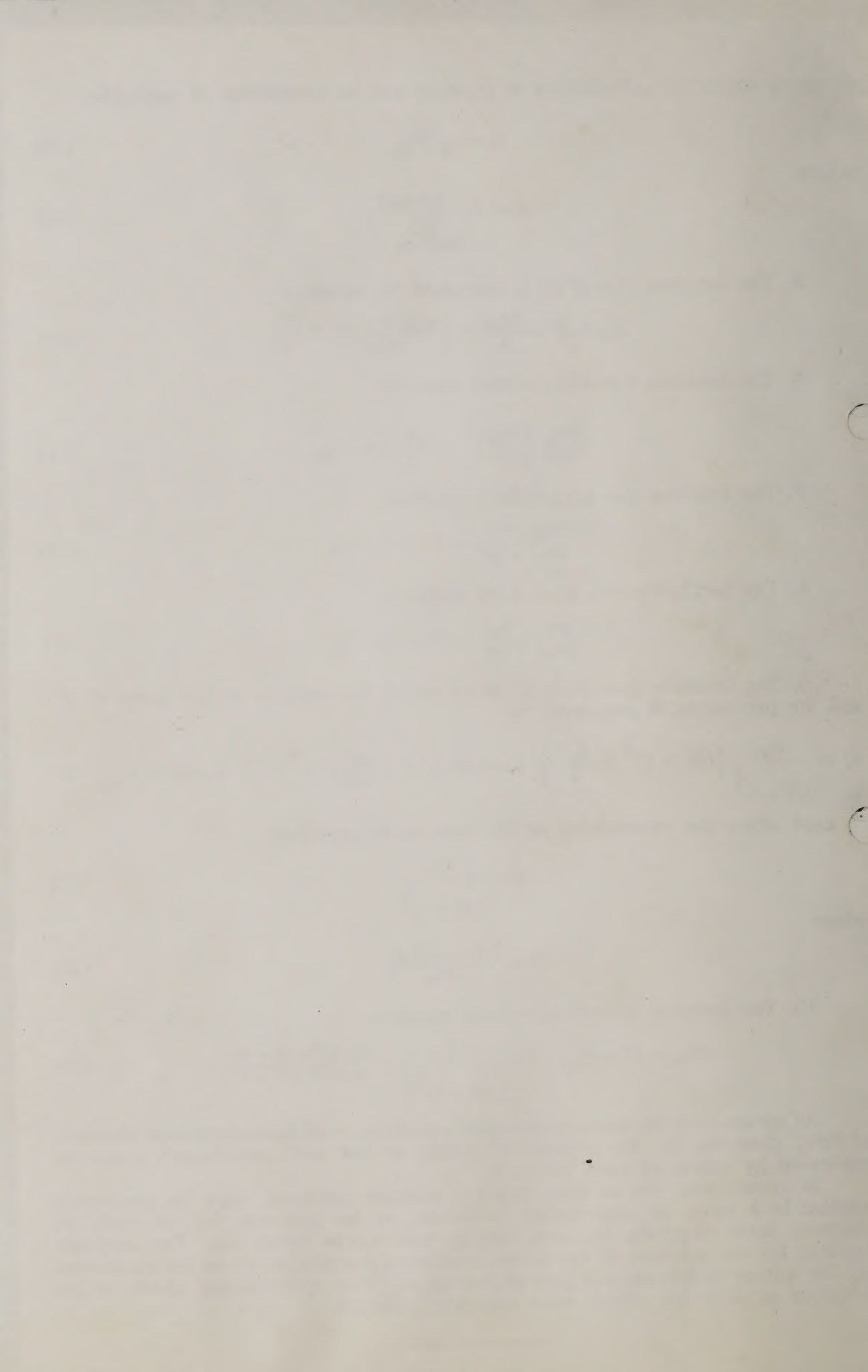
$$M = \frac{3(s-s_0)n_{00}^2}{2dr_{100}}; \quad (23)$$

10. The function $H = H(s)$, — from equation

$$H_{00} - H = H_{00} \left(1 - \frac{1}{(M+1)^{\frac{2}{3}}} \right) + \frac{H_{00} r_{10}^2 u_{00} (s-s_0)}{u_0 r_{10}^2 s_0^2 (M+1)}. \quad (24)$$

As we see from the above enumerated equations, — all hydrodynamical elements of flow, changing in the process of silting of the soil „structures“, — can be expressed by means of finite functions.

In conclusion, let us note, that the solution obtained, may be practically applied in a series of engineering problems, as for instance, in the study of weeping hole clogging, reservoir silting, waterworks filters etc. The methods suitable for the solution of the above mentioned practical problems are considered by the author in the second part of the paper. The experimental check of the obtained solution has shown most satisfactory results.



Trans. of Sci. Research Inst.
of Hydrotechnics, Vol. 16, 1935
Pressure Flow of Ground Water Carrying Fine Sandy and Clayey Particles.

By A. N. Patrashev, Eng.

Part I. Silting of Soil Skeletons.

This is a continuation of the paper published in Vol. XV of the Transactions of the Scientific Research Institute of Hydrotechnics.

The present paper contains a study of the silting criterion (Chapter III) and comparison of the theoretical solution (Chapters I, II, III) with experimental results obtained in laboratory and in the field (Chapter IV).

The combination of hydrodynamical elements of the stream and of conditions of the motion, which occur simultaneously with the process of silting, — constitute what we will denote the „siling criterion“.

We will, further, assume that two values of the silting criterion may exist: the lower value and the upper one. The lower value of the silting criterion corresponds to such a combination of hydrodynamical elements of the flow, where the diameter of solid particles (reduced to the spherical shape), is equal to the diameter of imaginary tubes through which percolating water travels in the skeleton of the soil.

The upper value of the silting criterion corresponds to such a combination of hydrodynamical elements of the flow, where the unstable process of sedimentation of solid particles in the interstices of the soil skeleton, begins to take place. Considering the flow as kinematically uniform and taking into account that before the beginning of the silting process the flow is uniform and laminar, we may determine the velocity of a solid particle from the following equation:

$$u_p = \frac{\gamma I_0}{4\mu} d^2 \left(\frac{3\eta_0 - 2}{6} \right) \quad (1)$$

Studying further the dynamic conditions of the motion of a solid particle and considering forces acting on it in the zones „a“ and „b“ (see fig. 16), we may conclude that a solid particle can be deposited by the stream only within the zone „a“; the length of this zone is:

$$\Delta S_a = \frac{\pi d}{6\rho_0} \eta_0 \quad (2)$$

Having further in view that the separation of a solid particle from the stream constitutes a breaking of inner ties and that a deposited particle is motionless, we may conclude that the moment when the accumulated energy of resistance of a solid particle is equal to its kinetic energy, — corresponds to the moment when the particle separates from the stream.

Then, using the law of kinetic energy we can obtain the upper value of the silting criterion as follows:

$$\eta'' = \frac{\psi + \sqrt{\psi^2 - 4\varphi\theta}}{2\varphi} \quad (3)$$

Thus, conditions determining the commencement of the silting, may be formulated as follows:

$$\eta' \leq \eta < \eta'' \quad (4)$$

As may be seen from definition of the symbols

$$\eta' = 1. \quad (5)$$

Experimental checking of the obtained solution has been performed on two installations; one of them represented a properly designed model of a percolation tube where zones of deposition of solid particles were observed by means of a photo-camera. The second model consisted of a horizontal flume with a box shaped partition, where a pressure flow through the tested soil skeleton, was created; the upper pool of the flume was supplied by means of a pump, by water saturated by fine sandy and clayey particles.

This installation served for measurement of hydrodynamical elements and their changes in time and along the path of the flow.

Comparison of results obtained by means of theoretical considerations with those derived from tests, shows that the scheme of flow as assumed in theoretical considerations coincides fairly well with that really observed.

Shape of functions assumed for calculation of various elements of the flow also agree quite satisfactorily with test results, as may be seen from the diagrams.

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PRESSURE FLOW OF GROUND WATER CARRYING
FINE SANDY AND CLAYEY PARTICLES.

By

A. N. PATRASHEV

in

Transactions of the Scientific Research
Institute of Hydrotechnic, Vol.XV,1935,
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TRANSLATION #69

by

S. N. Nasledov.

UNIVERSITY OF MICHIGAN
Department of Mechanical Engineering

REPORT

STRESSING PLAS ON THERMAL STRESS
IN THE JAW AND BONY TISSUES

BY

A. M. LARSEN

IN

Transactions of the Scientific Institute of Technology, Vol. 1, 1924, pp. 1-12

REPORT

RESEARCH

BY

A. M. LARSEN

UNIVERSITY OF CALIFORNIA

Berkeley

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Pressure flow of ground water carrying fine
sandy and clayey particles.

The present work was accomplished in the group of engineering hydraulics under direction of acad. N. N. Pavlovsky and has been reported in Leningrad on conference about reconstruction of river Volga (3/26/34).

The problem under consideration has great practical importance besides theoretical interest.

Indeed, such questions of hydrotechnical designs as: deposition of silt in reservoir, deposition of silt in up-stream side of dams, deposition of silt in filter of dams, in city water pipes, filtering openings of dams, forcing the particles of ground through filters, etc., as we know are almost not enlightened at all, not only from theoretical point of view but also from experimental point: But as it is known to practitioners, from the correct and rational solution of the latter (in by no means seldom cases) depends on the fate of whole hydrotechnical construction.

*(There is work with absolutely different solution of this question by S. V. Gbush (see Vol. X, 1933, of this transaction) and also experimental work of Gougentobler, Eng.)

In the present paper is given the theoretical solution of one of particular cases of movement of the stream saturated with small sandy and clayey particles, and work consists of two parts.

In first part-considered deposition of silt in ground skeletons by pressure, rectilinear "in main movement" stream;" and as an example here are given: Design for deposition of silt in filtrational openings of dams, and for deposition of silt of the filters in city water pipes. But since the last question is very vast, so in the present paper the principle design is given only with some additional suppositions.

*(First part consists of 8 chapters; first two of them are given here.)

In second part is considered forcing out of the small particles of ground from the ground skeletons.

1. The elements of the stream and of the process of deposition of silt.

Considering the differential elements of the stream, saturated with small sandy and clayey particles, we point out that such stream in respect to its physical properties is the heterogeneous stream; in respect to kinetic properties in general case, it is heterogeneous system of material points conditioned by difference in velocities of the movements of solid and liquid particles of it; in respect to dynamic properties in relation to transmission of movements from one particle of it to another, it is not different

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mission of movements from one particle of it to another, it is not different

from homogeneous liquid stream. In regard to dynamic conditions of such stream on surfaces of inner resistances or on walls, it is known that the latter are determined by physical and kinematic properties of it.

Turning to the consideration of the movement of such stream in homogeneous permeable ground medium, we note that the practice of hydrotechnical construction and specially performed experiments in hydrotechnical and filtration laboratories of scientific research, Institute of Hydrotechnics in 1933 and 1934, show that at corresponding values of N_0 and N_0 (see 3) occurs separation of solid particles, carried by stream, along way of movement; the stream, in passage through ground medium "cleared out", the solid particles partly or completely separated from the stream, and settle down in the pores of the ground medium.

Therefore, the stream, carrying solid particles, performed separation of them in the pores ground medium: on account of this, it is obviously noted that the area of cross-section of stream is diminishing, and also, therefore, the area of "entrance" of it into ground medium. But since this deposition process in time and along the way of the movement of the stream occurs not in the same way, so all hydrodynamic elements of it are also changed.

Such process of variation of water permeable property of ground skeletons usually called "deposition of silt."

of

2. Settling up the problem: The model movement. The solution of problem of movement of the stream with above-mentioned properties in most general cases of its movement, even in homogeneous permeable ground medium, is difficult for us right now.

In the present paper, we will consider particular case, the movement of such stream that is under pressure, stream-line "in main movement" rectilinear movement of it.

(See N. N. Pablovsky, Theory of movement of ground water under hydrotechnical constructions - Chapter 3, Part 1).

Besides, we assume that solid particles carried by ground stream in direction of the movements of it, have the same velocities of movements as the liquid particles should have which are equal to volumes of solid particles. Here we note that this agreement (condition) and in the conditions of stream-line regime of ground stream in relation of coarseness of carried solid particles which have place in practical problems of hydrotechnic, is not considerably limited. Besides, we also assume that in any given cross-section of stream, the quantity of solid particles located in the cross-section are distributed over cross-section area uniformly.

According to given below analysis, separation of solid particles from the stream, occurring in ground skeleton at the process of deposition of silt are not the result of its "free" precipitation since, because relatively rapid

from homogeneous liquid stream. In regard to dynamic conditions of such stream on surfaces of inner resistance or on walls, it is known that the latter are determined by physical and kinematic properties of it.

Turning to the consideration of the movement of such stream in homogeneous porous medium, we note that the practice of hydrotechnical construction and specially performed experiments in hydrotechnical and filtration laboratories of scientific research, Institute of Hydrotechnical Engineering, show that at corresponding values of K_0 and K_1 (see 3) occurs separation of solid particles, carried by stream, along way of movement; the stream, in passage through porous medium "cleans out", the solid particles partly or completely separated from the stream, and settle down in the pores of the ground medium.

Therefore, the stream, carrying solid particles, performed separation of them in the pores of ground medium: on account of this, it is obviously noted that the area of cross-section of stream is diminishing, and also, therefore, the area of "entrance" of it into ground medium. But since this deposition process in time and along the way of the movement of the stream occurs not in the same way, so all hydrodynamic elements of it are also changed.

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(See N. N. Fedorov, Theory of movement of ground water under hydrotechnical construction - Chapter 3, Part 1).

Besides, we assume that solid particles carried by ground stream in direction of the movement of it, have the same velocities of movement as the liquid particles should have which are equal to velocities of stream. Here we note that this assumption (condition) and in the conditions of stream-line regime of ground stream in relation of consistency of carried solid particles which have place in practical problems of hydrotechnical, is not completely limited. Besides, we also assume that in any given cross-section of stream, the quantity of solid particles located in the cross-section are distributed over cross-section area uniformly.

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change of velocities of stream at the bathing of separate grains of ground, the movement of solid particles along the parabola, as it takes place in "free" precipitation, is impossible.

Besides, according to given below analysis the separation from the stream of solid particles conditioned by difference in resistances to movement of solid and liquid particles of the stream, from the side, of the bathing by them ground medium and as the result of it the solid particles separate from the stream from outside layers of the stream and at time separate considerably earlier (see Ch. IV) than that takes place in "free" precipitation of them, particularly in the conditions of ground stream; it is quite proved by practice of hydrotechnical construction and especially by performed experiments; for this sufficiently to note here that maximal deposition of the grounds takes place always near their pressure surfaces, whereas in the presence of the "free precipitation it must be opposite. Therefore, the model stream, saturated with small sandy and clayey particles in respect to kinetic, is homogeneous system of material points subordinated to the same forces as separately taken moving liquid.

Considering the movement of such stream is homogeneous permeable ground medium, according to the principle of Professor N. N. Pavlovsky, we compose the model of the movement only in main, reflecting very complicated process of phenomena.

change of velocity of stream at the bottom of stream of ground, the movement of solid particles along the parabola, as it takes place in "free" precipitation, is impossible.

Second, according to given below analysis the separation from the stream of solid particles conditioned by difference in resistance to movement of solid and liquid particles of the stream, from the side, of the bottom of them around motion and as the result of it the solid particles separate from the stream from outside layers of the stream and at the same time separate from the stream (see Ch. IV) then that takes place in "free" precipitation of them, particularly in the conditions of ground stream; it is quite proved by practice of hydrotechnical construction and especially by numerous experiments; for this sufficiently to note here that maximal deposition of the grounds takes place always near their pressure surface, whereas in the presence of the "free" precipitation it must be opposite. Therefore, the model stream, saturated with sand and clay particles in respect to kinetic, is homogeneous system of material points subordinated to the same forces as separately taken moving liquid.

Considering the movement of such stream is homogeneous particles ground motion, according to the principle of Professor K. S. Lavrovsky, we compose the model of the movement only in water, reflecting very complicated process of phenomena.

In our case the model of movement may be represented by system of curvilinear percolating tubes the area of cross-section which constantly changing as in time as length of them. (See Fig. 1)*

On Figure 1 is plan of the model of the stream's movement. Dotted line shows configuration of main movement of stream; in places where centrifugal forces of inertia developed by moving stream are directed along inner normal, shown separated from the stream and deposited in the pores of permeable medium are the solid particles, on Fig. 1 shown only initial movement of deposition of silt in ground skeleton.

Indeed, if we are considering stream-line motion of stream homogeneous with respect to kinematic, then the separation from it of solid particles may occur only from outside layer of it, conditioned probably by different character of the resistance to the motion of solid and liquid particles which gives them ground medium; the resistances of the ground medium to the stream is surface forces of fraction; on the basis of existing laws of friction it is quite natural

to consider that the resistance to the motion of solid particles is the forces of friction between wetted solid bodies and the resistances to the motion of liquid particles is Newton's forces of friction in the liquids.

Normal forces (natural) because of existance of which the forces of friction takes place between wetted solid particles, in considered case, are centrifugal forces of inertia arising in the moving stream. Obviously, that these forces of friction will arise only there where centrifugal forces of inertia are directed along inner normal of curvature of the stream that is where the solid particles will pressed by the stream to the ground medium. Therefore, these forces of friction will arise not along whole wetted perimeter of percolation tube but only in part of it.

Besides in the less saturated with solid particles stream, they obviously will not influence to considerable extent on value of the losses of energy of movement of the stream. Therefore, the character of the de-



Фиг. 1. План модели движения. Траектории движения твердых частиц и места их отложений.

Figure 1.

*The plan model of movement. Trajectories of movement of solid particles and places of their deposition.

In our case the model of movement may be represented by a system of curvilinear parallel tubes the area of cross-section which constantly changes as in time as length of them. (See Fig. 1)*

On Figure 1 is a plan of the model of the stream's movement. Dotted line shows configuration of main movement of stream; in places where centrifugal forces of inertia developed by moving stream are directed along inner normal, shown separated from the stream and deposited in the pores of permeable medium are the solid particles, on Fig. 1 shown only initial movement of deposition of silt in ground skeleton.

Indeed, if we are considering stream-line motion of stream homogeneous with respect to kinematics, then the separation from it of solid particles may occur only from outside layer of it, conditioned probably by different character of the resistance to the motion of solid and liquid particles which gives them ground medium; the resistance of the ground medium to the stream is surface forces of friction; on the basis of existing laws of friction it is quite natural

Figure 1. The plan model of movement. Trajectories of movement of solid particles and places of their deposition.

to consider that the resistance to the motion of solid particles is the forces of friction between wetted solid bodies and the resistance to the motion of liquid particles is Newton's forces of friction in the liquids.

Normal forces (natural) because of existence of which the forces of friction takes place between wetted solid particles, in considered case, are centrifugal forces of inertia arising in the moving stream. Obviously that these forces of friction will arise only there where centrifugal forces of inertia are directed along inner normal of curvature of the stream that is where the solid particles will be pressed by the stream to the ground medium. Therefore, these forces of friction will arise not along whole wetted perimeter of permeation tube but only in part of it.

Besides in the case saturated with solid particles stream, they obviously will not influence to considerable extent on value of the losses of energy of movement of the stream. Therefore, the character of the de-

position of solid particles in main will be determined by values of centrifugal forces of inertia arising in the moving stream.

But together with it, it has to be kept in mind the presence arising forces of adhesion at touching separating from stream solid particles the wetted surface of ground medium.

Such in general the physics of phenomenon of the deposition of silt in ground skeletons. Here, also, we note that given above model of motion of stream was checked experimentally on special setup about this will more detailed talk in part of experimental rechecking of theoretical deductions.

From all above presented is not difficult to see that considered case of the movement of the stream is two-dimensional unsteady motion of the system of material points. Further, we will operate only with one percolation tube keeping in mind that in considered case the integration of the equation for whole region of ground medium is quite simple.

If the motion of the stream is curilinear as for example the movement under hydrotechnical constructions and the law of its movement before deposition of silt is known then and in this case the transition to the deposited with silt condition will not be complicated.

3. The main notations and some consequences from them.

Introduce the following notations:

Q Discharge of "basic" movement of stream through percolation tube.

u Average velocities of "basic" movement of stream in percolation tube.

Q Area of cross-section of stream = to area of percolation tube.

D Diameter of the percolation tube.

d Diameter of solid particles, taken as spheres carried by stream.

n % of solid particles in the unit volume of stream.

p Quantity of solid particles deposited on wetted perimeter of percolation tube, on its length equal d .

k Quantity of solid particles in given cross-section of stream.

ω Area of cross-section of solid discharge

$\eta = \frac{D}{d}$ coefficient of deposition of silt.

γ Weight of unit volume of the stream.

γh Piezometric pressure in given point of stream.

z The ordinates of the centre of gravity of the percolation tube counted

position of solid particles in water will be determined by values of certain forces of inertia arising in the moving stream.

But together with it, it has to be kept in mind the pressure arising forces of adhesion at touching separating from stream solid particles the wetted surface of ground medium.

Such is general the physics of phenomenon of the deposition of solid particles in stream. Here, also, we note that given above model of action of stream as checked experimentally on special setup about this will more detailed talk in part of experimental checking of theoretical deductions.

From all above presented is not difficult to see that considered case of the movement of the stream in two-dimensional unsteady motion of the system of material points. Further, we will operate only with one point-lattice keeping in mind that in considered case the interaction of the separating for whole region of ground medium is quite simple.

If the motion of the stream is unsteady as for example the movement under hydrodynamic conditions and the law of its movement before deposition of solid is known then and in this case the transition to the deposited with this condition will not be complicated.

3. The main notations and some consequences from them.

Introduce the following notations:

- X. φ Discharge of "basic" movement of stream through percolation tube.
- $\bar{v}$ Average velocities of "basic" movement of stream in percolation tube.
- $\bar{v}_c$ Area of cross-section of stream = to area of percolation tube.
- B. Diameter of the percolation tube.
- d. Diameter of solid particles, taken as spheres carried by stream.
- n. $\bar{v}$ of solid particles in the unit volume of stream.
- p. Quantity of solid particles deposited on wetted surface of percolation tube, on its length equal d.
- k. Quantity of solid particles in given cross-section of stream.
- m. Area of cross-section of solid discharge.
- $\gamma = \frac{d}{B}$ Coefficient of deposition of solid.
- Y Weight of unit volume of the stream.
- W. Piezometric pressure in given point of stream.
- Z The ordinate of the centre of gravity of the percolation tube counted

R. Radius of curvature of stream.

m. Mass of single solid particle.

H. Pressure in given point of stream.

τ Coefficient of friction between wetted solid bodies.

ρ Density of stream equal to

From given notations, it is useful to write the following relatives:

$$w = w_0 K \quad (1)$$

where $w_0 = \frac{\pi d^2}{4}$ (2)

this is from one side from another

$$w = \pi Q \quad (3)$$

therefore $K = \frac{\pi Q}{w_0}$ (4)

or $K = \pi Q^2$ (5)

Now we start to form differential relations which is mathematical expression above given the physics of the phenomena of deposition of silt.

CHAPTER I

The basic differential relations.

4. The equation of dynamic equilibrium for an element of the tube.

Cutting in the stream part of percolation tube by the length and for given moment of the time write its condition of dynamic equilibrium.

Applying to this part of tube the principle of D'Alembert, according to which the possible work of the loosed forces is equal to 0, in location of axes as on Figure 2, we have

$$M \frac{du}{dt} = S \quad (6)$$

Considering the Figure 2, it is not difficult to see that the mass of the element of the tube may be expressed as:

$$M = \rho \cdot Q \cdot \Delta S \quad (7)$$

Projection on the axes of the resultant of all outside forces acted upon the element of the tube will be composed of:

1. From projection of the force of the gravity of the volume equal to:

$$F_1 = \gamma \cdot Q \Delta s \frac{dz}{ds} \quad (8)$$

2. From projection of the hydrodynamic pressures in considered case expressed as:

$$\Delta P = P_1 Q_1 - P_2 Q_2 \quad (9)$$

or accepting that $P = P(s)$ and $Q = Q(s)$ are function (continuous) and neglecting variables of second order, we have

$$dP = - \left(Q \frac{dP}{ds} + P \frac{dQ}{ds} \right) \quad (10)$$

or remembering that $P = \gamma h$ equation (10) rewrites as:

$$dP = - \left(\gamma Q \frac{dh}{ds} + \gamma h \frac{dQ}{ds} \right) \quad (11)$$

where the changing γ along the line of stream we neglect, that is assuming that $\frac{d\gamma}{ds} = 0$

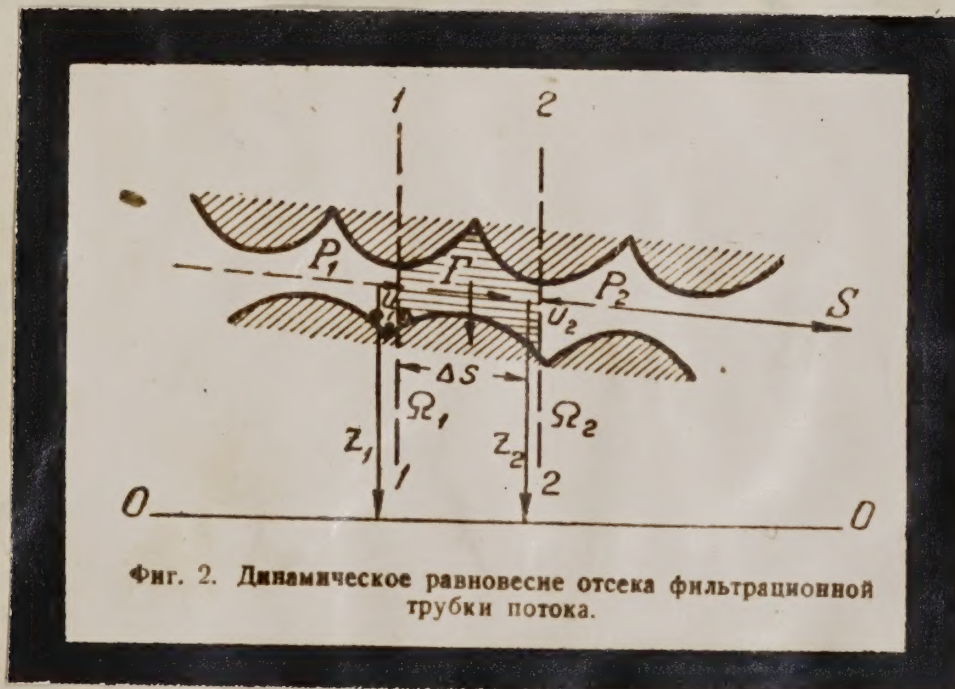


Figure 2.
Dynamic equilibrium of the element of the percolation tube of the stream.

3. And finally, from projection of the resultant of surface forces of friction.

Keeping in mind mentioned in we have that the projection of the friction's forces arose between solid particles carried by stream and the resistance to their motion by the ground medium may be expressed as

$$F_2 = -\tau \cdot \frac{\pi u^2}{R} \cdot \rho \cdot \frac{\pi \Delta s}{d} \quad (12)$$

projection of the friction's forces arose between liquid particles and the resistance to their motion by the ground medium determines according to Newton's law and may be expressed as:

$$F_3 = -\mu \frac{du}{dr} \Delta s X \quad (13)$$

(See N. N. Pavlovsky Hydraulics, part 1, 1928)

Summing up all members in one line and dividing both parts of equation on $\gamma Q \Delta s$ we finally receive equation as follows:

$$\frac{\rho}{\gamma} \frac{du}{dt} = -\frac{dz}{ds} + \frac{\partial h}{\partial s} - \frac{h}{Q} \frac{\partial Q}{\partial s} - \frac{\tau \pi u^2 \pi \rho}{\gamma R Q} \frac{\pi \rho}{d} - \frac{4\mu}{\gamma} \frac{du}{dr} \left(\frac{1}{d}\right) \quad (14)$$

5. The equation of the change of specific energy taking place along the length of the percolating tube.

Write the equation of Bernoulli for given moment of time for two cross-sections of percolation tube at distance from each other at Δs as it is shown on Figure 2.

We have:

$$z_1 + h_1 + \frac{u_1^2}{2g} = z_2 + h_2 + \frac{u_2^2}{2g} + h_i + h_w \quad (15)$$

or in differential from it may be written as:

$$\frac{\rho}{\gamma} \frac{\partial u}{\partial t} + \frac{d}{ds} \left(\frac{u^2}{2g} \right) = -\frac{dz}{ds} + \frac{\partial h}{\partial s} - \frac{\partial h_w}{\partial s} \quad (16)$$

or $\frac{\rho}{\gamma} \frac{du}{dt} = -\frac{dz}{ds} + \frac{\partial h}{\partial s} - \frac{\partial h_w}{\partial s}$
where Δh_w specific energy loosed by stream in the section Δs .

Here we stress the following condition: In considered case, the total energy in the cross-section 2-2 of stream, generally speaking, is somewhat greater than that in cross-section 1-1, because through cross-section 2-2 passed out somewhat larger discharge of the stream; but also because of that in the section Δs the sources of energy are absent, and the direction of the stream from cross-section 1-1 to 2-2 is the same, we come to the conclusion that specific energy of the stream in the cross-section 1-1 of the element of percolation tube must be greater than specific energy in cross-section 2-2.

2. and finally, from projection of the resultant of surface forces of friction.

Keeping in mind mentioned in we have that the projection of the friction's forces across between solid particles carried by stream and the resistance to their motion by the ground medium may be expressed as

(12) $F = - \gamma \cdot \frac{m v^2}{\rho} \cdot \rho \cdot \frac{m v^2}{\rho}$

projection of the friction's forces across between liquid particles and the resistance to their motion by the ground medium determined according to Newton's law and may be expressed as:

(13) $F = - \gamma \frac{d v}{d x}$

(See N. S. Pavlovsky Hydraulics, part I, 1938)

Summing up all members in one line and dividing both parts of equation on $\gamma Q d x$ we finally receive equation as follows:

(14) $\frac{1}{\gamma} \frac{d \gamma}{d x} = - \frac{d z}{d x} + \frac{d v}{d x} - \frac{v}{g} \frac{d v}{d x} - \frac{v^2}{2g} \frac{d v}{d x} - \frac{v^2}{2g} \frac{d v}{d x}$

3. The equation of the change of specific energy taking place along the length of the percolating tube.

Write the equation of Bernoulli for given moment of time for two cross-sections of percolation tube at distance from each other at Δx as it is shown on Figure 2.

We have:

(15) $E_1 + H_1 + \frac{v_1^2}{2g} = E_2 + H_2 + \frac{v_2^2}{2g} + H_3 + h_w$

or in differential form it may be written as:

(16) $\frac{1}{\gamma} \frac{d \gamma}{d x} + \frac{d H}{d x} + \frac{d v}{d x} \left(\frac{v}{g} \right) = - \frac{d z}{d x} + \frac{d v}{d x} - \frac{v}{g} \frac{d v}{d x} - \frac{v^2}{2g} \frac{d v}{d x}$

where Δh specific energy losses by stream in the section Δx .

Now we stress the following condition: In considered case, the total energy in the cross-section 2-2 of stream, generally speaking, is somewhat greater than that in cross-section 1-1, because through cross-section 2-2 passed out somewhat larger discharge of the stream; but also because of the in the section the sources of energy are absent, and the direction of the stream from cross-section 1-1 to 2-2 is the same, so come to the conclusion that specific energy of the stream in the cross-section 1-1 of the element of percolation tube must be greater than specific energy in cross-section 2-2.

Neglecting the loss of energy due to heat and kinetic energy separating from the stream the solid particles, the loosening by the stream energy in section Δs will be composed only of out of works of surface forces of friction and out of potential energy separating from the stream the solid particles in this section.

Then, the loosed-by-stream energy will be expressed as:

$$\Delta E = (F_2 + F_3 + \gamma h p w_0 \frac{\pi \Delta s}{d}) \Delta s \quad (18)$$

where F_2 and F_3 are determined by equations (12) and (13) obviously the specific energy loosed by stream will be:

$$\Delta h_w = \frac{1}{\gamma q \cdot \Delta s} (F_2 + F_3 + \gamma h \cdot p w_0 \frac{\pi \Delta s}{d}) \Delta s \quad (19)$$

Therefore,

$$\frac{\partial h_w}{\partial s} = 4 \frac{\mu}{\gamma} \frac{du'}{dr} \frac{1}{D} + \frac{r \mu u^2}{\gamma \cdot R \cdot q \cdot d} \cdot \pi p + h \cdot \frac{\pi p w_0}{dq} \quad (20)$$

Substituting value of $\frac{\partial h_w}{\partial s}$ into eq. (17), we finally get:

$$\frac{p}{\gamma} \frac{du}{dt} = - \frac{dz}{ds} + \frac{\partial h}{\partial s} - \frac{r \cdot \mu \cdot u^2}{\gamma R q d} \cdot \pi p - h \frac{\pi p w_0}{dq} - 4 \frac{\mu}{\gamma} \frac{du'}{dr} \frac{1}{D} \quad (21)$$

6. The equation showing the change of discharge along the length of percolation tube.

Let in given moment of time through cross section 1-1 (See Fig. 2) of percolation tube passes the discharge of the stream equal to q_1 , then at the same moment of time through cross section 2-2 located on distance Δs from cross section 1-1, the discharge will pass equal q_2 since in considered case takes place two dimensional unsteady motion.

Assuming that $q = q(s)$ is continuous function, the increase of discharge of the stream in given moment of time in the element will be equal to:

$$\Delta q = \frac{\partial q}{\partial s} \cdot \Delta s \quad (22)$$

But this increase of the discharge in the element Δs conditioned only by separation of solid particles from the stream since positive or negative sources of discharge here are absent.

The discharge of separating solid particles equal to:

$$q' = + p w_0 u \frac{\pi \Delta s}{d} \quad (23)$$

Then equating (22) and (23) get:

$$\frac{\partial q}{\partial s} + \frac{\pi p w_0}{d} \cdot u = 0 \quad (24)$$

neglecting the loss of energy due to heat and kinetic energy separation from the stream the solid particles, the loss of energy in section 4 will be composed only of out of work of surface forces of friction and out of potential energy separating from the stream the solid particles in this section.

Then, the loaded-stream energy will be expressed as:

$$(18) \quad \Delta E = (F_1 + F_2 + \gamma \rho_w g \frac{V}{A}) \Delta z$$

where F_1 and F_2 are determined by equations (12) and (13) obviously the specific energy loss by stream will be:

$$(19) \quad \Delta h = \frac{1}{\gamma} \Delta E = \frac{1}{\gamma} (F_1 + F_2 + \gamma \rho_w g \frac{V}{A}) \Delta z$$

Therefore,

$$(20) \quad \frac{dh}{dz} = \frac{1}{\gamma} \frac{dE}{dz} = \frac{1}{\gamma} \left(\frac{dF_1}{dz} + \frac{dF_2}{dz} + \gamma \rho_w g \frac{dV}{dz} \right)$$

Substituting value of $\frac{dE}{dz}$ into eq. (19), we finally get:

$$(21) \quad \frac{dh}{dz} = - \frac{dh}{dz} + \frac{dh}{dz} - \frac{dh}{dz} - \frac{dh}{dz} - \frac{dh}{dz} - \frac{dh}{dz} - \frac{dh}{dz} - \frac{dh}{dz} - \frac{dh}{dz} - \frac{dh}{dz}$$

8. The equation showing the change of discharge along the length of percolation tube.

Let in given moment of time through cross section 1-1 (see fig. 2) of percolation tube passes the discharge of the stream equal to Q_1 , then at the same moment of time through cross section 2-2 located on distance Δz from cross section 1-1, the discharge will pass equal to Q_2 since in considered case takes place two dimensional unsteady motion.

Assuming that $Q = Q(z)$ is continuous function, the increase of discharge of the stream in given moment of time in the element will be equal to:

$$(22) \quad \Delta Q = \frac{dQ}{dz} \cdot \Delta z$$

But this increase of the discharge in the element is conditioned only by separation of solid particles from the stream since positive or negative sources of discharge here are absent.

The discharge of separating solid particles equal to:

$$(23) \quad Q' = \gamma \rho_w g \frac{V}{A} \Delta z$$

Then equating (22) and (23) get:

$$(24) \quad \frac{dQ}{dz} = \gamma \rho_w g \frac{V}{A}$$

The same equation is not difficult to obtain from the conditions of compatible motion of solid and liquid particles.

7. The equation of continuity of the motion of the stream.

Turning to the Figure 2, it is not difficult to see that through cross section 1-1 of percolation tube in element of time Δt the volume of discharge will pass equal to:

$$V_1 = q_1 \Delta t \quad (25)$$

at the same element of time Δt through cross section 2-2 at a distance Δs from cross section 1-1 passing volume of stream will be:

$$V_2 = q_2 \Delta t \quad (26)$$

Then an accumulation of the volume of stream in element Δs equal to:

$$\Delta V = \frac{\partial q}{\partial s} \cdot \Delta s \cdot \Delta t \quad (27)$$

But because of continuity of movement of the stream (absence of the emptiness in it) this accumulation of the volume must be compensated, obviously by the change of the cross-sections of the stream in this section; increase Δs of volume will be

$$\Delta V = \frac{\partial q}{\partial t} \cdot \Delta t \cdot \Delta s \quad (28)$$

Equating expressions (27) and (28) and performing some simplification, we get well known equation of Saint-Venant derived by him for two dimensional unsteady motion

$$\frac{\partial q}{\partial s} + \frac{\partial q}{\partial t} = 0 \quad (29)$$

(See Outline of water flows according to Byssineskov.)

8. The equation of the change of head at a given point of the stream (The equation of dynamic equilibrium in the direction perpendicular to the motion of the stream.)

Take in the stream the element of percolation tube of the length equal to unity and write condition continuity in time of the direction "basic" motion of it. (See Figure 3.)

For this on the axis of the element of percolation tube separate element of area $s-s$; the condition of continuity of direction of "basic" motion of the stream, obviously will correspond to that acting normal forces upon area $s-s$ in whole motion will be equilibrated. To the forces changing direction of "basic" motion of the stream in considered case, are belonging: the weight of separating particle from the stream and increase of total piezometric pressure in given cross-section of the stream related only to the area of separated part of the stream in this cross-section. Then the

The same equation is not difficult to obtain from the conditions of compatibility of motion of solid and liquid particles.

7. The equation of continuity of the motion of the stream.

Turning to the figure 2, it is not difficult to see that through cross section I-I of percolation tube in element of time Δt the volume of discharge will pass equal to:

(25)

$$V = \Delta S \cdot \Delta t$$

at the same element of time Δt through cross section II-II at a distance Δx from cross section I-I passing volume of stream will be:

(26)

$$V = \Delta S \cdot \Delta t$$

Then an accumulation of the volume of stream in element Δx equal to:

(27)

$$\Delta V = \Delta S \cdot \Delta x \cdot \Delta t$$

But because of continuity of movement of the stream (absence of the empty mass in it) this accumulation of the volume must be compensated, obviously by the change of the cross-sections of the stream in this section; increase ΔS of volume will be

(28)

$$\Delta V = \Delta S \cdot \Delta x \cdot \Delta t$$

Replacing expressions (27) and (28) and performing some simplification, we get well known equation of Saint-Venant derived by him for two dimensional unsteady motion

(29)

$$\frac{\partial S}{\partial t} + \frac{\partial (Sv)}{\partial x} = 0$$

(See Outline of water flows according to Ilyashenko).

8. The equation of the change of head at a given point of the stream (The equation of dynamic equilibrium in the direction perpendicular to the motion of the stream).

Take in the stream the element of percolation tube of the length equal to unity and write condition continuity in time of the direction "basic" motion of it. (See figure 3.)

For this on the axis of the element of percolation tube separate element of area Δs ; the condition of continuity of direction of "basic" motion of the stream, obviously will correspond to that acting normal forces upon area Δs in whole motion will be equilibrium. To the forces belonging: direction of "basic" motion of the stream in considered case, are belonging: the weight of separating particle from the stream and increase of total piezometric pressure in given cross-section of the stream related only to the area of separated part of the stream in this cross-section. Then the

projection on axes OY of the weight of separated volume of the stream in element of time Δt in unit of stream equal to:

$$P_1 = - \gamma \Delta q \cdot \Delta t \cdot l \cdot \cos \alpha \cdot n \quad (30)$$

and the projection of "excess" of piezometric pressure upon area s-s in the same moment of time Δt equal to:

$$P_2 = \gamma \cdot \Delta h \cdot \Delta q \cdot l \cdot \cos \alpha \quad (31)$$

Equating (30) and (31) and dividing both parts of the equation by

$(\gamma \cdot \Delta q \cdot \Delta t)$ and taking the limits at $\Delta t \rightarrow 0$ get:

$$\frac{dh}{dt} + n \frac{dq}{dq} = 0 \quad (32)$$

or knowing that $q = u \cdot \rho$ rewriting as:

$$\frac{dh}{dt} + n \left(u + \rho \frac{\partial u}{\partial \rho} \right) = 0 \quad (33)$$

But having in mind that piezometric pressure in given point of the stream may be expressed as:

$$h = a + H_0' \quad (34)$$

where a is pressure from downstream side (in considered case "a" is constant) and equation (32) rewrites as:

$$\frac{dH}{dt} + n \cdot \frac{dq}{dq} = 0 \quad (35)$$

Finally, we have to note that obtained relation may be used with success in the hydraulics and in hydrodynamics of surface stream of liquid.

9. The equation of the deposition of solid particles considering the displacement of a volume of the stream equal to $V_1 = q \cdot l$ in element of time Δt through the length ds .

Then, because of the depositions of solid particles in permeable ground medium the volume of separated particles in element of time Δt through the length ds will equal to

$$W = \Delta q \cdot \Delta t \quad (36)$$

The same increase of the volume may be expressed through the number of solid particles carried by displaced volume, it will equal to:

$$W_1 = \omega_0 \cdot \Delta K \cdot n \cdot ds \quad (37)$$

Equating (36 and 37) and dividing both parts of equation on Δt we have

$$\Delta q = \omega_0 \frac{\Delta K}{\Delta t} n ds \quad (38)$$

projection on axes OY of the weight of separated volume of the stream in element of time Δt with of stream equal to:

$$(30) \quad P = \gamma \cdot \Delta z \cdot \Delta x \cdot n$$

and the projection of "excess" of piezometric pressure upon axes $z-z$ in the same moment of time Δt equal to:

$$(31) \quad P_z = \gamma \cdot \Delta z \cdot \Delta x \cdot n$$

Equating (30) and (31) and dividing both parts of the equation by

$$(32) \quad \gamma \cdot \Delta z \cdot \Delta x \cdot n \rightarrow 0$$

$$\frac{\partial h}{\partial z} + n \frac{\partial z}{\partial z} = 0$$

or knowing that $z = \alpha \cdot P$ rewriting as:

$$(33) \quad \frac{\partial h}{\partial z} + n \left(\alpha + P \frac{\partial \alpha}{\partial z} \right) = 0$$

but having in mind that piezometric pressure in given point of the stream may be expressed as:

$$(34) \quad h = z + h_p$$

where h_p is pressure from downstream side (in considered case "a" is constant) and equation (33) rewritten as:

$$(35) \quad \frac{\partial h}{\partial z} + n \cdot \frac{\partial z}{\partial z} = 0$$

Finally, we have to note that obtained relation may be used with success in the hydraulics and in hydrodynamics of surface stream of liquid.

3. The equation of the deposition of solid particles considering the displacement of a volume of the stream equal to $\gamma \cdot \Delta z \cdot \Delta x$ in element of time Δt through the length Δx .

Then, because of the deposition of solid particles in permeable ground medium the volume of separated particles in element of time Δt through the length Δx will be equal to

$$(36) \quad W = \gamma \cdot \Delta z \cdot \Delta x$$

The same increase of the volume may be expressed through the number of solid particles carried by displaced volume, it will equal to:

$$(37) \quad W_s = w_s \cdot \Delta x \cdot n \cdot \Delta z$$

Equating (36 and 37) and dividing both parts of equation on Δx we have

$$(38) \quad \gamma \cdot \Delta z = w_s \cdot \Delta z \cdot n$$

where ΔQ total increase of the discharge that is total differential whereas ΔK partial differential only of variable t , since separation of solid particles occurs only in plane of some diametral cross-section of the stream, independent of distance s at which it is located from the beginning of movements. Dividing both part of eq. (38) by dt and having in mind that $\frac{ds}{dt} = u$ get

$$\frac{dQ}{dt} = \pi w_0 u \cdot \frac{\partial K}{\partial t} \quad (39)$$

or

$$Q \cdot \frac{du}{dt} + u \frac{dQ}{dt} = \pi \cdot w_0 u \cdot \frac{\partial K}{\partial t} \quad (40)$$

10. Relation between n , p , and η .

Turning to the Fig. 4, let p' will be numbers of particles depositing along the radii of percolating tube in given moment of time.

Accordingly, described model of the motion is given moment of time along the radii of the tube and may deposit only one particle because deposition is occurred only in outside layer of the stream therefore.

$$p' = 1 \quad (41)$$

If the relation of wetted perimeter of percolation tube in which ξ in given moment of time occurs deposition of solid particles to the total perimeter of percolated tube will be represented as ξ then the number of separating solid particles along the total perimeter of the tube as the length of the stream equal d may be presented as:

$$p = \frac{\pi D}{d} p' n \xi \quad (42)$$

or substituting value of p' and remembering that

$$\frac{D}{d} = \eta \quad (43)$$

we get

$$p = \pi \xi \eta \quad (44)$$

where Δ total increase of the discharge that is total differential whereas
 Δ partial differential only of variable Δ , since separation of solid
 particles occurs only in plane of some diameter cross-section of the stream,
 independent of distance z at which it is located (the beginning of move-
 ments. Dividing both part of eq. (38) by Δz and having in mind that $\frac{\Delta z}{\Delta t} = v$ we get

$$(38) \quad \frac{\Delta \rho}{\Delta t} = n \cdot v \cdot \frac{\Delta \rho}{\Delta z}$$

$$(40) \quad \frac{\Delta \rho}{\Delta t} + v \cdot \frac{\Delta \rho}{\Delta z} = n \cdot v \cdot \frac{\Delta \rho}{\Delta z}$$

10. Relation between n , v , and ρ .

Turning to the Fig. 4, let ρ will be number of particles depositing along
 the radii of percolation tube in given moment of time.

Accordingly, described model of the motion is given moment of time along
 the radii of the tube and may deposit only one particle because deposition
 is occurred only in outside layer of the stream therefore.

$$(41) \quad \rho = 1$$

If the relation of wanted perimeter of percolation tube in which ρ in given
 moment of time occurs deposition of solid particles to the total perimeter
 of percolated tube will be represented as ρ then the number of separating
 solid particles along the total perimeter of the tube as the length of the
 stream equal ρ may be presented as:

$$(42) \quad \rho = \frac{\pi \cdot d \cdot \rho}{\pi \cdot d \cdot \rho}$$

or substituting value of ρ and remembering that

$$(43) \quad \rho = \frac{\pi \cdot d \cdot \rho}{\pi \cdot d \cdot \rho}$$

$$(44) \quad \rho = \pi \cdot d \cdot \rho$$

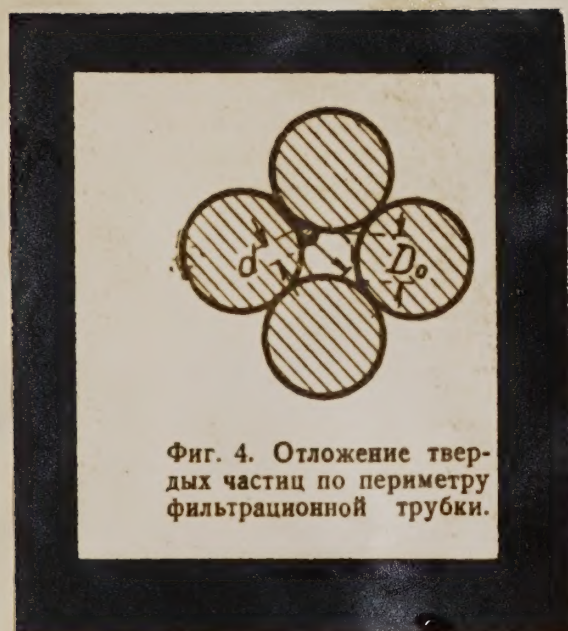


Figure 4.
The deposition of solid particles along the perimeter of percolation tube.



Figure 5.
Determination of radii of the curvature of the stream.

Having in mind described model of the motion, it is not difficult to show that ξ is constant in all time of movement of the stream and equal to $\frac{1}{\pi}$; here we note that for the flat stream $\xi = 0.25$. Indeed, ξ shows that element of wetted perimeter of percolation tube where the centrifugal forces of inertia arising in stream directed along inner normal; for cylindrical tube this element of perimeter equal to diameter of it and for the square equal to the side of it. Then for ρ we get following expression

$$\rho = \pi \eta \quad (45)$$

11. The equation of change of the cross-section of the solid discharge.

The change of the cross-section of the solid discharge easily may be expressed through the number of solid particles, separating in the pores of ground medium:

$$\Delta W = + W_0 \rho \Delta \eta \quad (46)$$

This relation is obvious and therefore does not require any proofs, it may also be written:

$$\frac{\partial W}{\partial t} - W_0 \pi \eta \frac{\partial \eta}{\partial t} = 0 \quad (47)$$

Besides, if we accept that $w = w(s)$ is continuous function (that quite justified) then it is not difficult to receive the following relation:

$$\frac{\partial w}{\partial s} - W_0 \pi \eta \frac{\partial \eta}{\partial s} = 0 \quad (48)$$

12. The equation of the radii of the curvature of the stream.

From simple geometrical considerations given on Figure 5, it is easily shown that-

$$R = R_0 \left[\cos \varphi + \sin \varphi \frac{\cos \varphi + \sin \varphi}{\cos \varphi - \sin \varphi} \right] \quad (49)$$

where

$$R_0 = 1.25 D_0 \quad (50)$$

$$\frac{\sin \varphi}{2} = 1.2 \left(1 - \frac{\eta}{\eta_0} \right) \quad (51)$$

Therefore, substituting values of $\sin \varphi$ and $\cos \varphi$ to the eq. (49) we get:

$$R = R_0 \left[1 - 2a^2 + 2a^2 \sqrt{1-a^2} \cdot \frac{1-2a^2(1-\sqrt{1-a^2})}{1-2a^2(1+\sqrt{1-a^2})} \right] \quad (52)$$

where

$$a = 1.2 \left(1 - \frac{\eta}{\eta_0} \right) \quad (53)$$

Obtained equation can be considerably simplified not only algebraically, but also by neglecting small member of higher power. But in many practical cases, there is no necessity because the force of friction arising between solid particles and ground media at small value of η influences very little the value of the loss of the pressure along the length; and the equation

of the radii of the curvature is never included in other differential equations, therefore inaccuracy in its determination cannot be reflected upon other relations.

13. Additional remarks: For further mathematical operation, it is very useful to write the following relations:

1. Since
$$u = \frac{Q}{\varrho} \quad (54)$$

then
$$\frac{\partial u}{\partial t} = \frac{1}{\varrho} \frac{\partial Q}{\partial t} - \frac{Q}{\varrho^2} \frac{\partial \varrho}{\partial t} = \frac{1}{\varrho} \left(\frac{\partial Q}{\partial t} - u \frac{\partial \varrho}{\partial t} \right) \quad (55)$$

remembering $\frac{\partial \varrho}{\partial t} = -\frac{\partial \varrho}{\partial s}$ where we get

$$\frac{\partial u}{\partial t} = \frac{1}{\varrho} \frac{dQ}{dt} \quad (56)$$

2. Since
$$\omega = nQ = \omega_0 n \eta^2 \quad (\text{see eqn. 3}) \quad (57)$$

then
$$\frac{\partial \omega}{\partial t} = \omega_0 \left(\eta^2 \frac{\partial n}{\partial t} + 2 n \eta \frac{\partial \eta}{\partial t} \right) \quad (58)$$

and also
$$\frac{\partial \omega}{\partial s} = \omega_0 \left(\eta^2 \frac{\partial n}{\partial s} + 2 n \eta \frac{\partial \eta}{\partial s} \right) \quad (59)$$

3. And finally, relations with respect to the stream

$$\begin{aligned} Q(t) &= u(t) \cdot \varrho(t) \\ Q(s) &= u(s) \cdot \varrho(s) \end{aligned} \quad (60)$$

14. Conclusions: Therefore, it is not difficult to see that obtained system of differential equations may be reduced to the system of Jacob's differential equations that will be accomplished in the next paragraph.

Chapter 2.

Integration of the system of the differential equations.

1. Reducing of the system of the differential equations to Jacob's system and establishing of mutual relations between the elements of the stream.

15. Relation between n and η as function of time. Substituting the value of $\frac{\partial \omega}{\partial t}$ into eq. (47) we get:

$$\omega_0 \left(\eta^2 \frac{\partial n}{\partial t} + 2 n \eta \frac{\partial \eta}{\partial t} \right) - \omega_0 n \eta \frac{\partial \eta}{\partial t} = 0 \quad (61)$$

performing possible simplification:

$$\eta \frac{\partial n}{\partial t} + n \frac{\partial \eta}{\partial t} = 0 \quad (62)$$

using the Jacob's method get:

$$\boxed{\eta = C_1 n} \quad (63)$$

where C_1 , determined from the primary conditions is equal to:

of the nature of the curve is never included in other differential equations, therefore inaccuracy in its determination cannot be reflected upon other relations.

13. Additional remarks: For further mathematical operation, it is very useful to write the following relations:

1. Since

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial w}{\partial t} \cdot \frac{dt}{dt}$$

then

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial w}{\partial t} \cdot \frac{dt}{dt}$$

where

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial w}{\partial t} \cdot \frac{dt}{dt}$$

2. Since

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial w}{\partial t} \cdot \frac{dt}{dt}$$

then

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial w}{\partial t} \cdot \frac{dt}{dt}$$

and also

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial w}{\partial t} \cdot \frac{dt}{dt}$$

3. And finally, relations with respect to the stream

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial w}{\partial t} \cdot \frac{dt}{dt}$$

4. Conclusions: Therefore, it is not difficult to see that obtain-
 of system of differential equations may be reduced to the system of Jacob's
 differential equations that will be accomplished in the next paragraph.

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial w}{\partial t} \cdot \frac{dt}{dt}$$

Chapter 2.

Integration of the system of the differential equations.

1. Reduction of the system of the differential equations to Jacob's
 system and establishing of mutual relations between the elements of the stream.

2. Relation between w and ψ as function of time. Substituting the
 value of $\frac{\partial w}{\partial t}$ into eq. (47) we get:

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial w}{\partial t} \cdot \frac{dt}{dt}$$

performing possible simplification:

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial w}{\partial t} \cdot \frac{dt}{dt}$$

Using the Jacob's method get:

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial w}{\partial t} \cdot \frac{dt}{dt}$$

where $\frac{\partial w}{\partial t}$ determined from the primary conditions is equal to:

$$C_1 = \frac{\eta_0}{\eta} \quad (63^1)$$

From it follows that the change of the diameter of percolation tube in given cross-section, in time, is directly proportional to the change of the % of concentration of the stream in the same cross-section at time.

16. Relation between u and η as function of time. Substituting value of $\frac{dw}{dt}$ from eq. (39) into eq. (56) get:

$$\frac{du}{dt} = \frac{w_0}{Q} \eta \cdot \frac{d\eta}{dt} u \quad (64)$$

remembering that

$$w = w_0 \eta \quad (65)$$

therefore

$$w_0 \frac{d\eta}{dt} = \frac{dw}{dt} \quad (66)$$

Then performing substitutions and simplifications rewrite eq. (64):

$$\frac{1}{u} \frac{du}{dt} = \frac{\eta}{Q} \frac{dw}{dt} \quad (67)$$

substituting value of $\frac{dw}{dt}$ from eq. (58):

$$\frac{1}{u} \frac{du}{dt} = \frac{\eta}{\eta^2} \left(\eta^2 \frac{d\eta}{dt} + 2\eta \eta \frac{d\eta}{dt} \right) \quad (68)$$

or

$$\frac{1}{u} \frac{du}{dt} = \frac{\eta}{\eta} \left(\eta \frac{d\eta}{dt} + 2\eta \frac{d\eta}{dt} \right) \quad (69)$$

or, having in mind eq. (62)

$$\frac{1}{u} \frac{du}{dt} = \frac{\eta^2}{\eta} \frac{d\eta}{dt} \quad (70)$$

but since

$$\eta = \frac{\eta}{C_1} \quad (71)$$

then

$$\frac{1}{u} \frac{du}{dt} = \frac{\eta}{C_1^2} \frac{d\eta}{dt} \quad (72)$$

and it is easy to see that it gives:

$$C_2 u = e^{\frac{\eta^2}{2C_1^2}} \quad (73)$$

where e is the base of natural logarithms C_2 determined from primary conditions is equal to:

$$C_2 = \frac{e^{\frac{\eta_0^2}{2}}}{u_0} \quad (74)$$

Expression (73) may be rewritten as:

$$\frac{u}{u_0} = e^{\frac{\eta_0^2}{2} \left(\frac{\eta^2}{\eta_0^2} - 1 \right)} \quad (75)$$

or solving eq. (73) with respect to η get:

or solving eq. (73) with respect to $\frac{dC}{dt}$:

$$\frac{dC}{dt} = C \left(\frac{1}{\tau} - 1 \right)$$

Expression (73) may be rewritten as:

$$C = \frac{C_0}{2}$$

where C_0 is the base of natural logarithm e determined from primary conditions is equal to:

$$C_0 = e^{\frac{1}{\tau}}$$

and it is easy to see that it gives:

$$\frac{1}{\tau} = \frac{1}{C} \frac{dC}{dt}$$

then

$$C = \frac{1}{\tau}$$

but since

$$\frac{1}{\tau} = \frac{1}{C} \frac{dC}{dt}$$

or, having in mind eq. (62)

$$\frac{1}{\tau} = \frac{1}{C} \left(\frac{1}{\tau} + \frac{1}{\tau} \right)$$

or

$$\frac{1}{\tau} = \frac{1}{C} \left(\frac{1}{\tau} + \frac{1}{\tau} \right)$$

substituting value of $\frac{dC}{dt}$ from eq. (58):

$$\frac{1}{\tau} = \frac{1}{C} \left(\frac{1}{\tau} + \frac{1}{\tau} \right)$$

Then performing substitutions and simplifications rewrite eq. (64):

$$\frac{1}{\tau} = \frac{1}{C} \left(\frac{1}{\tau} + \frac{1}{\tau} \right)$$

therefore

$$C = \frac{1}{\tau}$$

remembering that

$$\frac{1}{\tau} = \frac{1}{C} \left(\frac{1}{\tau} + \frac{1}{\tau} \right)$$

value of $\frac{dC}{dt}$ from eq. (59) into eq. (58) and:

18. Relation between C and τ as function of time. Substituting

of concentration of the stream in the cross-section at time, in time, is directly proportional to the change of the C from its value at the change of the diameter of percolation tube in time.

$$C = \frac{1}{\tau}$$

(65)

$$\eta^2 = 2 C_1^2 \ln(u \cdot C_2) \quad (76)$$

or

$$\eta^2 = \eta_0^2 \left(1 + \frac{2}{\eta_0^2} \ln \frac{u}{u_0}\right) \quad (77)$$

From where it is not hard to see that the diameter of percolation tube changes at time considerably more intensive than the velocity of the stream's movement in it.

We stress that this situation is very impressive since in considered case H_0 (pressure on constructions) is constant at all time of stream's movement.

17. Relation between u and η , as function of time. Substituting into eq. (72) value of η from eq. (63) get:

$$\frac{1}{u} \frac{du}{dt} = \eta \frac{d\eta}{dt} \quad (78)$$

Intergrating eq. (78):

$$u C_3 = e^{\frac{\eta^2}{2}} \quad (79)$$

where C_3 determined from primary conditions is equal to:

$$C_3 = \frac{e^{\frac{\eta_0^2}{2}}}{u_0} \quad (80)$$

Then eq. (79) may be rewritten:

$$\frac{u}{u_0} = e^{\frac{\eta^2 - \eta_0^2}{2}} \quad (81)$$

or, solving eq. (81) with respect η we have:

$$\eta^2 = \eta_0^2 + 2 \ln \frac{u}{u_0} \quad (82)$$

or

$$\left(\frac{\eta}{\eta_0}\right)^2 = 1 + \frac{2}{\eta_0^2} \ln \frac{u}{u_0} \quad (83)$$

or in general way:

$$\eta = + \sqrt{2 \ln(u C_3)} \quad (84)$$

Again the velocity of the stream movement at time changes considerably slower than that of % concentration of stream.

18. Relation between H and η as function of time write eq. (38) in such way:

$$\frac{\partial H}{\partial t} + \eta \left(u + \varrho \frac{\partial u}{\partial \varphi}\right) = 0 \quad (85)$$

instead of φ we introduce variable η

$$\varphi = \omega_0 \eta^2 \quad (86)$$

18. Relation between η and ϕ as function of time write eq. (55) in such way:

$$(55) \quad \frac{\partial \eta}{\partial t} + \eta \left(\eta + \frac{\partial \eta}{\partial \phi} \right) = 0$$

instead of ϕ we introduce variable η

$$(56) \quad \eta = \omega_0 \eta'$$

or in general way:

$$(57) \quad \left(\frac{\eta}{\omega_0} \right)' = 1 + \frac{\eta}{\omega_0} \ln \frac{\eta}{\omega_0}$$

$$(58) \quad \eta' = \omega_0' + 2 \ln \frac{\eta}{\omega_0}$$

or, solving eq. (51) with respect η we have:

$$(59) \quad \boxed{\frac{\eta}{\omega_0} = e^{\frac{\eta' - \omega_0'}{2}}}$$

where ω_0 determined from primary conditions is equal to:

$$(60) \quad \omega_0 = e^{\frac{\omega_0'}{2}}$$

Interpreting eq. (52):

$$(61) \quad \frac{1}{\eta} \frac{\partial \eta}{\partial t} = \eta \frac{\partial \eta}{\partial \phi}$$

into eq. (52) value of ϕ from eq. (53) for:

17. Relation between η and τ , as function of time. Substituting

case H_0 (pressure on constrictions) is constant at all time of stream's movement.

We stress that this situation is very impressive since in considered

from where it is not hard to see that the diameter of constriction tube changes at time considerably more intensive than the velocity of the stream's movement in it.

$$(62) \quad \eta' = \omega_0' \left(1 + \frac{2}{\omega_0'} \ln \frac{\eta}{\omega_0} \right)$$

$$(63) \quad \eta' = 2 \omega_0' \ln \left(\frac{\eta}{\omega_0} \right)$$

$$\varrho \frac{\partial u}{\partial \varrho} = \frac{\eta}{2} \frac{\partial u}{\partial \eta} \quad (87)$$

Then eq. (85) takes form:

$$\frac{\partial H}{\partial t} + \left(u + \frac{\eta}{2} \frac{\partial u}{\partial \eta} \right) \eta = 0 \quad (88)$$

The value of $\frac{\partial u}{\partial \eta}$ is easy to determine from eq. (73):

$$\frac{\partial u}{\partial \eta} = e^{\frac{\eta^2}{2C_1^2}} \cdot \frac{\eta}{C_1^2 C_2} \quad (89)$$

or

$$\frac{\eta}{2} \frac{\partial u}{\partial \eta} = e^{\frac{\eta^2}{2C_1^2}} \cdot \frac{\eta^2}{2C_1^2} \quad (90)$$

but according to eq. (73)

$$\frac{e^{\frac{\eta^2}{2C_1^2}}}{C_2} = u \quad (91)$$

then eq. (88) becomes:

$$\frac{\partial H}{\partial t} = - \left(1 + \frac{\eta^2}{2C_1^2} \right) \cdot u \cdot \eta \quad (92)$$

solving simultaneously eq. (24) and (29):

$$+ \frac{\partial \varrho}{\partial t} = \frac{\eta \cdot p \cdot \omega_0}{d} \cdot u \quad (93)$$

or substituting the value of p from eq. (45) and $\frac{\partial \varrho}{\partial t}$ from eq. (86) have:

$$2 \omega_0 \eta \frac{\partial \eta}{\partial t} = \eta^2 \eta \frac{\omega_0}{d} \cdot u \quad (94)$$

simplifying:

$$2 d \frac{\partial \eta}{\partial t} = \eta^2 u \quad (95)$$

Therefore

$$u = \frac{+2d \frac{\partial \eta}{\partial t}}{\eta^2} \quad (96)$$

Substituting the value of u from eq. (96):

$$\frac{\partial H}{\partial t} = - \frac{\eta \left(1 + \frac{\eta^2}{2C_1^2} \right) d \frac{\partial \eta}{\partial t}}{C_1 \frac{\eta^2}{2C_1^2}} \quad (97)$$

or

$$\frac{\partial H}{\partial t} = - \left(\frac{2dC_1}{\eta} + \frac{d\eta_0 \eta}{\eta_0} \right) \frac{\partial \eta}{\partial t} \quad (98)$$

Integral of this equation is

$$+ H = - \left(\frac{2d\eta_0}{\eta_0} \ln \eta + \frac{d\eta_0 \eta^2}{2\eta_0} \right) + C_4 \quad (99)$$

Substituting the value for independent constant C_4 which is found by satis-

fyng with primary conditions, have:

$$H_0 - H = \frac{2d\eta_0}{\eta_0} \ln \frac{\eta_0}{\eta} + \frac{d\eta_0}{2\eta_0} (\eta_0^2 - \eta^2) \quad (100)$$

But having in mind that the values of d and η_0 are very small (see Part III of eq. 39) we may, with accuracy enough for the practical purposes, express eq. (100) as follows:

$$H_0 - H = \frac{2d\eta_0}{\eta_0} \ln \frac{\eta_0}{\eta} \quad (101)$$

or solving eq. (101) with respect η we have

$$\eta = \eta_0 e^{-\frac{(H_0 - H)\eta_0}{2d\eta_0}} \quad (102)$$

19. Relation between H and n as function of time. Substituting value of n instead of η from eq. (63) into eq. (97) we have:

$$\frac{dH}{dt} = - \frac{2d(1 + \frac{n^2}{2})}{n} \cdot C_1 \frac{dn}{dt} \quad (103)$$

Integral of this equation is:

$$+H = - \frac{2d\eta_0}{\eta_0} \ln - \frac{d\eta_0}{\eta_0} \cdot \frac{n^2}{2} + C_5 \quad (104)$$

Substituting value of the constant of the integration we have:

$$H_0 - H = \frac{2d\eta_0}{\eta_0} \ln \frac{\eta_0}{\eta} + \frac{d\eta_0}{2\eta_0} (\eta_0^2 - \eta^2) \quad (105)$$

But having in mind the considerations about eq. (100) we rewrite eq. (105) as:

$$H_0 - H = \frac{2d\eta_0}{\eta_0} \ln \frac{\eta_0}{\eta} \quad (106)$$

or solving it with respect to n :

$$n = \eta_0 e^{-\frac{(H_0 - H)\eta_0}{2d\eta_0}} \quad (107)$$

20. Relation between H and u as function of time. Rewriting using substitutions from eq. (73) and (72)

$$\frac{1}{2d} \frac{dH}{dt} = - \frac{[1 + \ln(uC_3)]}{\sqrt{2 \ln(uC_3)}} \cdot C_1 \frac{1}{u \sqrt{2 \ln(uC_3)}} \cdot \frac{du}{dt} \quad (108)$$

If we denote

$$\ln(uC_3) = x \quad (109)$$

then eq. (108) becomes:

$$\frac{dH}{d} = - \frac{\eta_0}{\eta_0} \frac{(1+x)dx}{x} \quad (110)$$

Integral of eq. (110) is

$$H = - \frac{d\eta_0}{\eta_0} \ln x - \frac{d\eta_0}{\eta_0} x + C_6' \quad (111)$$

lying

with primary conditions, have:

(100)

$$H_0 - H = \frac{2dH_0}{n} \ln \frac{H_0}{H} + \frac{dH_0}{2n} (H_0^2 - H^2)$$

But having in mind that the values of d and n_0 are very small (see Part III of ed. 32) we may, with accuracy enough for the practical purposes, express ed. (100) as follows:

(101)

$$H_0 - H = \frac{2dH_0}{n} \ln \frac{H_0}{H}$$

(102)

or solving ed. (101) with respect to H we have

$$H = H_0 e^{-\frac{(H_0 - H)n}{2dH_0}}$$

19. Relation between H and n as function of time. Substituting value of n instead of n' from ed. (63) into ed. (97) we have:

(103)

$$\frac{dH}{dt} = - \frac{2d(H_0 - H)}{n} \cdot C \cdot \frac{dH}{dt}$$

Integral of this equation is:

(104)

$$H - H_0 = - \frac{2dH_0}{n} \ln \frac{H_0 - H}{H_0} + C_1$$

Substituting value of the constant of the integration we have:

(105)

$$H_0 - H = \frac{2dH_0}{n} \ln \frac{H_0}{H_0 - H} + \frac{dH_0}{2n} (H_0^2 - H^2)$$

But having in mind the considerations about ed. (100) we rewrite ed. (105) as:

(106)

$$H_0 - H = \frac{2dH_0}{n} \ln \frac{H_0}{H}$$

or solving it with respect to n :

(107)

$$n = \frac{2dH_0}{H_0 - H} \ln \frac{H_0}{H}$$

20. Relation between H and n as function of time. Rewriting using substitutions from ed. (73) and (75)

(108)

$$\frac{1}{2d} \frac{dH}{dt} = - \frac{[1 + \ln(u^2)]}{\sqrt{2} \ln(u)} \cdot C \cdot \frac{1}{\sqrt{2} \ln(u^2)}$$

(109)

$$\ln(u^2) \cdot x$$

If we denote

then ed. (108) becomes:

(110)

$$\frac{dH}{dt} = - \frac{H_0}{K} (1+x) \ln x$$

Integral of ed. (110) is

(111)

$$H = - \frac{dH_0}{n} \ln x - \frac{dH_0}{n} x + C_1$$

Substituting values for constant of integration:

$$H_0 - H = \frac{d\eta_0}{\eta_0} \left[\ln \frac{x_0}{x} + (x_0 - x) \right] \quad (112)$$

Having in mind eq. (109) and eq. (80) we rewrite eq. (112) as follows:

$$H_0 - H = \frac{d\eta_0}{\eta_0} \left[\ln \left(\frac{\eta_0^2}{\eta_0^2 + 2 \ln \frac{u}{u_0}} \right) + \ln \frac{u_0}{u} \right] \quad (113)$$

Considering obtained equation, it is easy to see that we may neglect the second member in bracket because at $\ln u/u_0 = \frac{\eta_0^2}{2}$. The first member of equation is equal to infinity. After this, simplification eq. (113) becomes

$$H_0 - H = \frac{d\eta_0}{\eta_0} \ln \left(\frac{\eta_0^2}{\eta_0^2 + 2 \ln \frac{u}{u_0}} \right) \quad (114)$$

or solving this equation with respect to u we have

$$u = u_0 e^{\frac{\eta_0^2}{2} \left(\frac{1}{(H_0 - H) \eta_0} \frac{d\eta_0}{d\eta_0} - 1 \right)} \quad (115)$$

Considering eq. (115) we note the following:

1. With considerable change of the pressure at time, the velocity of the stream is practically changed very little; for the streams with concentration not over 10% with sufficient accuracy, we may accept that the change of velocity at time is not dependable upon change of pressure at time; at the same time, we stress that here we have in mind the change of functions only as functions of time, that is, all the rest of the arguments pertaining to these functions are supposed, in these cases, to be constants and enter into the latter in form of parameters.

2. From eq. (115) follows that to negative increase of the pressure corresponds also negative increase of the velocity of time.

21. Relation between n and η as function of the distance transmitted s . For obtaining relation n from η as $\frac{dw}{ds}$; that is, assuming all arguments to be their parameters, it is sufficient to substitute value of $\frac{dw}{ds}$ from eq. (59) into eq. (48) after that we have:

$$w_0 \left(\eta^2 \frac{d\eta}{ds} + 2\eta \eta \frac{d\eta}{ds} \right) - w_0 \eta \eta \frac{d\eta}{ds} = 0 \quad (116)$$

Simplifying:

$$\eta \frac{d\eta}{ds} + \eta \frac{d\eta}{ds} = 0 \quad (117)$$

since the sign of $\frac{d\eta}{ds}$ and $\frac{d\eta}{ds}$ are different

$$\eta \frac{d\eta}{ds} - \eta \frac{d\eta}{ds} = 0 \quad (118)$$

Then using Jacob's method, we set up characteristic equation and solving it:

$$\boxed{\eta \cdot \eta = C_6} \quad (119)$$

where

$$C_6 = \eta_{00} \eta_{00} ; \quad (120)$$

and the quantity η_{00} and η_{00} correspond to s_0 to the portion of the distance transmitted where the stream deposits the solid particles into the ground medium in given moment of time. Therefore, η_{00} and η_{00} are functions only in time, interlacing of which determined by eq. (63).

22. Relation between u and η as function of distance transmitted s . Subtracting from eq. (14), eq. (21) we have:

$$\frac{\eta}{\varphi} \frac{d\varphi}{ds} = \frac{\eta}{\varphi} \cdot \frac{\eta \cdot p \cdot \omega_0}{d} \quad (121)$$

or after simplifying

$$\frac{d\varphi}{ds} = \frac{\eta \cdot p \cdot \omega_0}{d} \quad (122)$$

moreover, we rewrite eq. (24) in following way:

$$\varphi \frac{du}{ds} + u \frac{d\varphi}{ds} + \frac{\eta \cdot p \cdot \omega_0}{d} \cdot u = 0 \quad (123)$$

having in mind eq. (120) we get:

$$\varphi \frac{du}{ds} + 2u \frac{d\varphi}{ds} = 0 \quad (124)$$

but since $\varphi = \omega_0 \eta^2$ and

$$\frac{d\varphi}{ds} = 2\omega_0 \eta \frac{d\eta}{ds} \quad (125)$$

then eq. (122) becomes:

$$\frac{1}{u} \frac{du}{ds} = - \frac{\eta}{\eta} \frac{d\eta}{ds} \quad (126)$$

Integrating, we have

$$\boxed{u \eta^4 = C_7} \quad (127)$$

where C_7 determined by primary conditions is equal to:

$$C_7 = u_{00} \eta_{00}^4 \quad (128)$$

where u_{00} and η_{00} correspond to S_0 in given moment of time.

It is not without interest to write (127) as follows:

$$\boxed{u \varphi^2 = C_8} \quad (129)$$

where

$$C_8 = u_{00} \varphi_{00}^2 \quad (130)$$

From this follows that for any moment of time and in any cross-section of the stream, the product of the velocity of the movement and square of the area of the cross-section of the percolation tube is constant quantity; equals to the product of the velocity of the stream's movement and square of the area of cross-section of percolation tube corresponding to S_0 the distance transmitted where the stream deposits the solid particles into ground medium.

23. Relation between u and η as function of S . Substituting value $\frac{\partial \eta}{\partial S}$ from eq. (117) into eq. (126) we get

$$\frac{1}{u} \frac{\partial u}{\partial S} = \frac{4}{\eta} \frac{\partial \eta}{\partial S} \quad (131)$$

Interating it:

$$\boxed{u = C_9 \eta^4} \quad (132)$$

where

$$C_9 = \frac{u_{00}}{\eta_{00}^4} \quad (133)$$

and u_{00} and η_{00} correspond to S_0 for given moment of time.

The relation (132) shows that for the given moment of time the change of the stream's velocity along its movement is proportional to fourth power of % concentration of the stream, which also changes along the movement of the stream.

24. Relation between H and η as function of S . Rewrite eq. (14) in the following way:

$$\frac{P}{\gamma} \frac{du}{dt} = -\frac{dz}{ds} + \frac{\partial h}{\partial s} - \frac{h}{Q} \frac{\partial Q}{\partial s} - \frac{\gamma m u^2}{\gamma R Q} \cdot \frac{m P}{d} - \frac{4M}{\gamma} \frac{1}{D} \frac{du}{dr} \quad (134)$$

or, having in mind that

$$Q = w_0 \eta^2 \quad (135)$$

$$\frac{P}{\gamma} \frac{du}{dt} = -\frac{dz}{ds} + \frac{\partial h}{\partial s} - \frac{2h}{\eta} \frac{\partial \eta}{\partial s} - \frac{2\gamma \cdot m u^2}{\gamma R w_0} \cdot \frac{1}{\eta} \frac{\partial \eta}{\partial s} - \frac{4M}{\gamma} \frac{1}{D} \frac{du}{dr} \quad (136)$$

If the plane of comparison is drawn on the level of the horizon of the down-stream side, then

$$H = h - z \quad (137)$$

where H piezometric pressure is given point of the stream from where:

$$\frac{\partial H}{\partial s} = \frac{\partial h}{\partial s} - \frac{\partial z}{\partial s} \quad (138)$$

if the angle of inclination toward the horizon of the percolation tube is equal to 0, then:

$$\frac{\partial H}{\partial s} = \frac{\partial h}{\partial s} \quad (139)$$

And according to it and third member of eq. (136) may be rewritten as follows:

$$\frac{2H}{\eta} \frac{\partial \eta}{\partial s} = \frac{2H}{\eta} \frac{\partial \eta}{\partial s} \quad (140)$$

Indeed, the member of equation $\frac{2H}{\eta} \cdot \frac{\partial \eta}{\partial s}$ characterizes the provision of the specific potential energy of deposited particles into the ground skeleton relatively to some independently chosen plane of the comparison; although it is also to obtain this immediately if eq. (9) is rewritten as follows:

$$\frac{\Delta P}{\gamma} = H_1 \rho_1 - H_2 \rho_2 \quad (141)$$

Then performing operations such as have been done in derivation of eq. (10), it is to obtain eq. (140).

Besides, since the velocities of stream-line movement of the stream are very small, the mass of the separating from the stream solid particles, in comparison to the mass of the stream, are also very small and the radii of the curvature of the stream with duration of time is increasing as it shows by eq. (52) so it appears to be possible to neglect the influence of the friction's forces (arising between solid particles and ground medium) on the magnitude of the losses of the pressure along the movement of the stream.

This statement is quite proven by the experiments and also it may easily be checked immediately from calculations; also, we note that this statement is not contradictory of the accepted in present paper model of the movement of percolating stream, because here we speak only of the losses of the pressure which has no direct relation to the cause of the deposition of the solid particles by the stream into ground medium.

Moreover, it should be borne in mind that the latter occurs in the section of way (flow), which is equal to only about ; this is more pronounced (noticeable) with the small percentage of the flow's concentration; yet in practice, the latter happens more frequently than anything else. Therefore, in respect to the losses of the pressure considered here problem reduces to the movement of the homogeneous stream; undoubtedly, it is assumption for the purpose of more simple mathematical result, but for usually occurring in the nature streams with the saturation not over 10% and lower-this assumption is within the limits of the accuracy of the calculations.

And, finally, having in mind equation determining form of the function $u = u(t)$ (See eq. 176), we will accept that:

$$\frac{\rho}{\gamma} \frac{du}{dt} = \frac{u}{g} \frac{du}{ds} \quad (142)$$

Having in mind above mentioned remark, we rewrite eq. (136) as follows:

$$\frac{u}{g} \frac{\partial u}{\partial s} = \frac{\partial H}{\partial s} - \frac{2H}{\eta} \frac{\partial \eta}{\partial s} - \frac{A \frac{M}{\gamma}}{d} \cdot \frac{du}{dr} \quad (143)$$

Multiply all members of the equation on η and having in mind eq. (126) and eq. (127), we have:

$$- \frac{4u_0^2 \eta_0^8}{g \eta^8} \cdot \frac{\partial \eta}{\partial s} = \eta \frac{\partial H}{\partial s} - 2H \frac{\partial \eta}{\partial s} - \frac{4 \frac{M}{\gamma}}{d} \cdot \frac{du}{dr} \quad (144)$$

or

$$\left(2H - \frac{4u_0^2 \eta_0^8}{9\eta^8}\right) \frac{\partial \eta}{\partial s} = \eta \frac{\partial H}{\partial s} - \frac{4\frac{M}{\eta}}{d} \cdot \frac{du}{dr} \quad (145)$$

It is easy to see that if $\frac{\partial \eta}{\partial s} = 0$ then

$$\frac{\partial H}{\partial s} = \frac{4\frac{M}{\eta}}{d\eta} \cdot \frac{du}{dr} \quad (146)$$

which is corresponded to the uniform stream-line motion of Poisseli and also to the law of Darcy. Further, accept that the law of the resistance of the motion corresponding to the uniform stream-line motion remains in force for, as considered here, nonuniform motion, the same as it takes place in the hydraulics of the nonuniform motion without pressure; in our problem, this has more basis since we consider there stream-line motion, the outside layer of which independently forms the character of the movement which has the same physical significance.

Therefore, the last member of the eq. (144) will express the gradient of the losses of the pressure corresponding to the resistances of movement on "walls", proportional to first power of the velocity and equal to:

$$\frac{4\frac{M}{\eta}}{d\eta} \cdot \frac{du}{dr} = \frac{32M}{\eta} \cdot \frac{u}{d^2} = \frac{32M}{\eta d^2} \cdot \frac{u}{\eta^2} \quad (147)$$

(See N. N. Pavlovsky, Hydraulics Part I, 1928, p. 251).

Denoting

$$\beta = \frac{32M}{\eta d^2} \quad (148)$$

then eq. (147) takes form:

$$\frac{4\frac{M}{\eta}}{d\eta} \cdot \frac{du}{dr} = \beta \frac{u}{\eta^2} \quad (149)$$

Having in mind (149), we rewrite eq. (145) as:

$$\frac{\partial H}{\partial s} = \left(\frac{2H}{\eta} - \frac{4u_0^2 \eta_0^8}{9\eta^9} \right) \frac{\partial \eta}{\partial s} + \beta \frac{u}{\eta^2} \quad (150)$$

or, remembering that

$$u = \frac{u_0 \eta_0^4}{\eta^4} \quad (151)$$

or

$$\frac{\partial H}{\partial s} = \left(\frac{2H}{\eta} - \frac{4u_0^2 \eta_0^8}{9\eta^9} \right) \frac{1}{\eta} \frac{\partial \eta}{\partial s} + \frac{\beta u_0 \eta_0^4}{\eta^6} \quad (152)$$

$$\frac{1}{H} \frac{\partial H}{\partial s} = - \left(1 - \frac{2u_0^2 \eta_0^8}{9 \cdot H \cdot \eta^8} \right) \frac{2}{\eta} \cdot \frac{\partial \eta}{\partial s} + \frac{\beta u_0 \eta_0^4}{H \eta^6} \quad (153)$$

25. Relation between H and η as functions of s.

Having in mind eq. (118), (119), and (120) and making substitutionⁿ of η , we have

$$\frac{1}{H} \cdot \frac{\partial H}{\partial s} = + \left(1 - \frac{2u_0^2}{9H} \frac{\eta^8}{\eta_0^8} \right) \frac{2}{\eta} \frac{\partial \eta}{\partial s} + \frac{\beta u_0}{H} \frac{\eta^6}{\eta_0^6 \eta_0^2} \quad (154)$$

26. Relation between H and u as function of s .
Rewrite eq. (143) as follows:

$$+ \frac{1}{H} \frac{\partial H}{\partial s} = \frac{2}{\eta} \frac{\partial \eta}{\partial s} - \frac{u}{\eta H} \frac{\partial u}{\partial s} + \frac{\beta u}{H \cdot \eta^2} \quad (155)$$

having in mind eq. (126) and (127), rewrite eq. (155) as:

$$+ \frac{1}{H} \cdot \frac{\partial H}{\partial s} = - \left(\frac{1}{2u} + \frac{u}{\eta H} \right) \frac{\partial u}{\partial s} + \frac{\beta}{u_{00}^{\frac{1}{2}} \eta_{00}^2} \cdot \frac{u^{\frac{3}{2}}}{H} \quad (156)$$

It is interesting to note that if we neglect the losses of the pressure against the friction, then we receive the following very simple relations:

$$\frac{H}{H_{00}} = \left(\frac{\eta_{00}}{\eta} \right)^2 \quad (157)$$

$$\frac{H}{H_{00}} = \left(\frac{u}{u_{00}} \right)^{\frac{1}{2}} \quad (158)$$

$$\frac{H}{H_{00}} = \left(\frac{\eta}{\eta_{00}} \right)^2 \quad (159)$$

Besides, considering the eq. (156) it is interesting to note that the change of the pressures along the length of the stream is proportional to the velocity of the stream at $\frac{3}{2}$ power; this condition points out that, together with the change of the cross-section area along its resistances to the motion are increasing. Here, we also note that this is not all in contradiction to the accepted in the present paper the law of resistances, because in the expression (149) is included, the variable quantity and depending on the velocity of the stream.

Therefore, we obtained twelve mutual relations between hydrodynamic elements of the stream changing as in time as along the way of the movement of it, which are quite sufficient for the finding (establishing) of the form of the functions for each of them from the same variables.

II. Establishing of the form of functions from variables t and s for individual hydrodynamic elements of the stream.

27. Equation, determining form of function $u = u(t)$. For determination function $u = u(t)$, we write relation (72) which has the following form:

$$\frac{1}{u} \frac{\partial u}{\partial t} = \frac{\eta}{C^2} \frac{\partial \eta}{\partial t} \quad (160)$$

but according to eq. (95)

$$\frac{\partial \eta}{\partial t} = \frac{\eta^2 u}{2d} \quad (161)$$

at the same time

$$\eta = \frac{\eta}{C} \quad (162)$$

Then

$$\frac{2d}{u^2} \cdot \frac{\partial u}{\partial t} = \frac{\eta^3}{C^4} \quad (163)$$

25. Relation between η and function of η .
 Rewrite eq. (143) as follows:

$$(143) \quad \frac{1}{4} \frac{\partial \eta}{\partial z} = \frac{1}{2} \frac{\partial \eta}{\partial t} - \frac{\eta}{2} \frac{\partial \eta}{\partial z} + \frac{\eta}{2} \frac{\partial \eta}{\partial t}$$

having in mind eq. (126) and (127), rewrite eq. (143) as:

$$(144) \quad \frac{1}{4} \frac{\partial \eta}{\partial z} = - \left(\frac{1}{2} \frac{\partial \eta}{\partial t} + \frac{\eta}{2} \frac{\partial \eta}{\partial z} \right) + \frac{\eta}{2} \frac{\partial \eta}{\partial t}$$

It is interesting to note that if we neglect the losses of the pressure against the friction, then we receive the following very simple relations:

$$(145) \quad \frac{\eta}{2} = \left(\frac{\partial \eta}{\partial t} \right)^2$$

$$(146) \quad \frac{\eta}{2} = \left(\frac{\partial \eta}{\partial z} \right)^2$$

$$(147) \quad \frac{\eta}{2} = \left(\frac{\partial \eta}{\partial t} \right)^2$$

Besides, considering the eq. (143) it is interesting to note that the change of the pressure along the length of the stream is proportional to the velocity of the stream at $\frac{1}{2}$ power; this condition points out that, together with the change of the cross-section area along its resistance to the motion are increasing. Here, we also note that this is not all in contradiction to the accepted in the present paper the law of resistance, because in the expression (143) is included, the variable quantity and depending on the velocity of the stream.

Therefore, we obtained two mutual relations between hydrodynamic elements of the stream changing as in time along the way of the movement of it, which are quite sufficient for the finding (establishing) of the form of the functions for each of them from the same variables.

II. Establishing of the form of function from variables t and z for individual hydrodynamic elements of the stream.

27. Section, determining laws of function η and η' for detailed relation function η and η' , we write relation (72) which has the following form:

$$(148) \quad \frac{1}{2} \frac{\partial \eta}{\partial z} = \frac{\eta}{2} \frac{\partial \eta}{\partial t} - \frac{\eta}{2} \frac{\partial \eta}{\partial z}$$

but according to eq. (92)

$$(149) \quad \frac{\partial \eta}{\partial z} = \frac{\eta}{2} \frac{\partial \eta}{\partial t}$$

$$(150) \quad \frac{\partial \eta}{\partial z} = \frac{\eta}{2} \frac{\partial \eta}{\partial t}$$

$$(151) \quad \frac{\partial \eta}{\partial z} = \frac{\eta}{2} \frac{\partial \eta}{\partial t}$$

at the same time

Then

but according to eq. (76)

$$\eta^2 = 2 C_1^2 \ln(u C_2) \quad (164)$$

Substituting from eq. (164) into eq. (163) and making some simplifications:

$$\frac{d C_1 du}{u^2 \ln(u C_2) \sqrt{\ln(u C_2)} \cdot \sqrt{2}} = dt \quad (165)$$

For integration of this equation, we make the following substitutions:

Let $\ln(u C_2) = x$ (166)

then $u = \frac{e^x}{C_2}$ (167)

$$\frac{du}{u} = dx \quad (168)$$

$$C_2 \frac{d C_1}{\sqrt{2}} \cdot \frac{dx}{e^x x^{\frac{3}{2}}} = dt \quad (169)$$

Then expanding function e^{-x} into series of power x , we have

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} \quad (170)$$

Since this series is rapidly convergent (See eq. 73) then for practical purposes, it is sufficient to take only first two members after which we get:

$$\frac{C_2 d C_1}{\sqrt{2}} \left[x^{-\frac{3}{2}} - x^{-\frac{1}{2}} \right] dx = dt \quad (171)$$

Integral of this equation is:

$$\frac{C_2 d C_1}{\sqrt{2}} \left(-\frac{2}{\sqrt{x}} - 2\sqrt{x} \right) = t + D, \quad (172)$$

Substituting the values of x from eq. (167) and of constant of the integration which determines from eq. (172) by primary conditions we get:

$$\frac{\sqrt{2} d e^{\frac{\eta_0^2}{2}} \cdot \eta_0}{u_0 \eta_0} \left[\frac{\sqrt{2}}{\sqrt{2 \ln \frac{u}{u_0} + \eta_0^2}} - \frac{\sqrt{2}}{\eta_0} + \left(\eta_0 \sqrt{2 \ln \frac{u}{u_0} + \eta_0^2} \right) \frac{1}{\sqrt{2}} \right] = t - t_0 \quad (173)$$

$$\text{or } \frac{2 d e^{\frac{\eta_0^2}{2}} \cdot \eta_0}{u_0 \eta_0} \left[\frac{\eta_0 - \sqrt{2 \ln \frac{u}{u_0} + \eta_0^2}}{\eta_0 \sqrt{2 \ln \frac{u}{u_0} + \eta_0^2}} + \frac{\eta_0 - \sqrt{2 \ln \frac{u}{u_0} + \eta_0^2}}{2} \right] = t - t_0 \quad (174)$$

Considering obtained equation, it is easy to see that second member in brackets is the quantity negligibly small in comparison with the first one; therefore, eq. (174) in final form may be written as:

$$\frac{2 d e^{\frac{\eta_0^2}{2}} \cdot \eta_0}{u_0 \eta_0^2} \cdot \frac{\eta_0 - \sqrt{2 \ln \frac{u}{u_0} + \eta_0^2}}{\sqrt{2 \ln \frac{u}{u_0} + \eta_0^2}} = t - t_0 \quad (175)$$

or

$$\frac{2 d e^{\frac{\eta_0^2}{2}} \eta_0}{u_0 \eta_0^2} \left(\frac{\eta_0}{\sqrt{2 \ln \frac{u}{u_0} + \eta_0^2}} - 1 \right) = t - t_0 \quad (176)$$

but according to eq. (76)

$$(164) \quad y'' = 2C^2 \ln(u, C)$$

Substituting from eq. (164) into eq. (163) and making some simplifications:

$$(165) \quad \frac{d^2 C}{dx^2} = \frac{2C^2 \ln(u, C)}{\sqrt{2C^2 \ln(u, C)}} = 2C$$

For integration of this equation, we make the following substitution:

$$(166) \quad \ln(u, C) = x$$

Let

$$(167) \quad u = \frac{C^2}{C}$$

then

$$(168) \quad \frac{du}{dx} = 2C$$

$$(169) \quad C^2 \frac{du}{dx} = 2C^3$$

Then expanding function e^x into series of power x , we have

$$(170) \quad e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \dots$$

Since this series is rapidly convergent (see eq. 78) then for practical purposes, it is sufficient to take only first two members since which we get:

$$(171) \quad \frac{C^2}{\sqrt{2}} \left[x - \frac{x^2}{2} \right] dx = 2C^3$$

Integral of this equation is:

$$(172) \quad \frac{C^2}{\sqrt{2}} \left(-\frac{x}{2} - \frac{x^2}{6} \right) = 2 + C$$

Substituting the value of x from eq. (167) and of constant of the integration which determines from eq. (172) by primary conditions we get:

$$(173) \quad \frac{C^2}{\sqrt{2}} \left[\frac{1}{\sqrt{2}} \ln \frac{u}{C} + \left(\frac{u}{C} - \frac{1}{\sqrt{2}} \right) \right] = 2 + C$$

$$(174) \quad \frac{C^2}{\sqrt{2}} \left[\frac{1}{\sqrt{2}} \ln \frac{u}{C} + \left(\frac{u}{C} - \frac{1}{\sqrt{2}} \right) \right] = 2 + C$$

Considering obtained equation, it is easy to see that second member in brackets is the quantity negligible small in comparison with the first one; therefore, eq. (174) in final form may be written as:

$$(175) \quad \frac{C^2}{\sqrt{2}} \ln \frac{u}{C} = 2 + C$$

or

$$u = u_0 e^{\frac{m_0^2}{2} \left(\frac{1}{\left[\frac{(t-t_0) u_0 m_0^2}{2 d e^{\frac{m_0^2}{2} q_0}} + 1 \right]^2} - 1 \right)} \quad \text{--- (176')}$$

28. Consideration of equation (176)

In eq. (176), we have to note the following

$$1. \text{ At } \ln \frac{u}{u_0} = \frac{m_0^2}{2} (t - t_0) = \infty \quad (177)$$

Therefore, the minimal value of the velocity of the stream as function of time is:

$$u_{\min} = \frac{u_0}{e^{\frac{m_0^2}{2}}} \quad (178)$$

and hence, the limits of the change of the velocity in time is:

$$u_0 \geq u \geq \frac{u_0}{e^{\frac{m_0^2}{2}}} \quad (179)$$

Having in mind that in practical problems the concentration of the stream is usually not higher than 10%, it should be accepted that the stream's velocity in time does not change to any extent; such deduction physically quite understandable, because in considered problem it is assumed that the primary (initial) pressure upon the constructions is constant--not changing in time.

Therefore, in considered problem the reserve of the specific potential energy of the stream entering into ground skeleton does not change in time, and that insignificantly small change of the velocity of the stream in time which is determined by eq. (176), conditioned by heterogeneousness of the stream.

Indeed, in our problem, the water particles only have the carrying ability; therefore, initial pressure, with forcing into the stream the solid particles (in respect to its carrying ability), must be correspondingly decreased. On the other hand, it is necessary to keep in mind that the velocity of the separating from the stream solid particles is not equal to 0, and it correspondingly changes the velocity of the movement of its neighbors in moment of its separation from the stream. As it appears to us, that is the cause of the change of velocity of the stream in time.

2. The greater the values of the initial velocity of stream and its concentration and the lesser the initial sectional area of the filtration pipe, the more intensive is the change of the stream's velocity in time.

And finally, for more obvious presentation of function we give its graphical interpretation. (See Figure 6).

22. Consideration of equation (175)

In eq. (175), we have to note the following

(177)
$$\lim_{t \rightarrow \infty} \frac{u}{u_0} = \frac{1}{2} (1 - \epsilon) = \infty$$

Therefore, the minimal value of the velocity of the stream as function of time is:

(178)
$$u_{min} = \frac{u_0}{2} (1 - \epsilon)$$

and hence, the limits of the change of the velocity in time is:

(179)
$$u_0 \geq u \geq \frac{u_0}{2} (1 - \epsilon)$$

Having in mind that in practical problems the concentration of the stream is usually not higher than 10%, it should be accepted that the stream's velocity in time does not change to any extent; such reduction physically quite understandable, because in considered problem it is assumed that the primary (initial) pressure upon the construction is constant--not changing in time.

Therefore, in considered problem the reserve of the specific potential energy of the stream entering into ground skeleton does not change in time, and that insignificantly small change of the velocity of the stream in time which is determined by eq. (175), conditioned by heterogeneity of the stream.

Indeed, in our problem, the water particles only have the carrying ability; therefore, initial pressure, with forcing into the stream the solid particles (in respect to its carrying ability), must be correspondingly decreased. On the other hand, it is necessary to keep in mind that the velocity of the separating from the stream solid particles is not equal to 0, and it correspondingly changes the velocity of the movement of its neighbors in moment of its separation from the stream. As it appears to us, that is the cause of the change of velocity of the stream in time.

3. The greater the values of the initial velocity of stream and its concentration and the lesser the initial sectional area of the filtration pipe, the more intensive is the change of the stream's velocity in time.

And finally, for more obvious presentation of function we give its graphical interpretation. (See Figure 5).

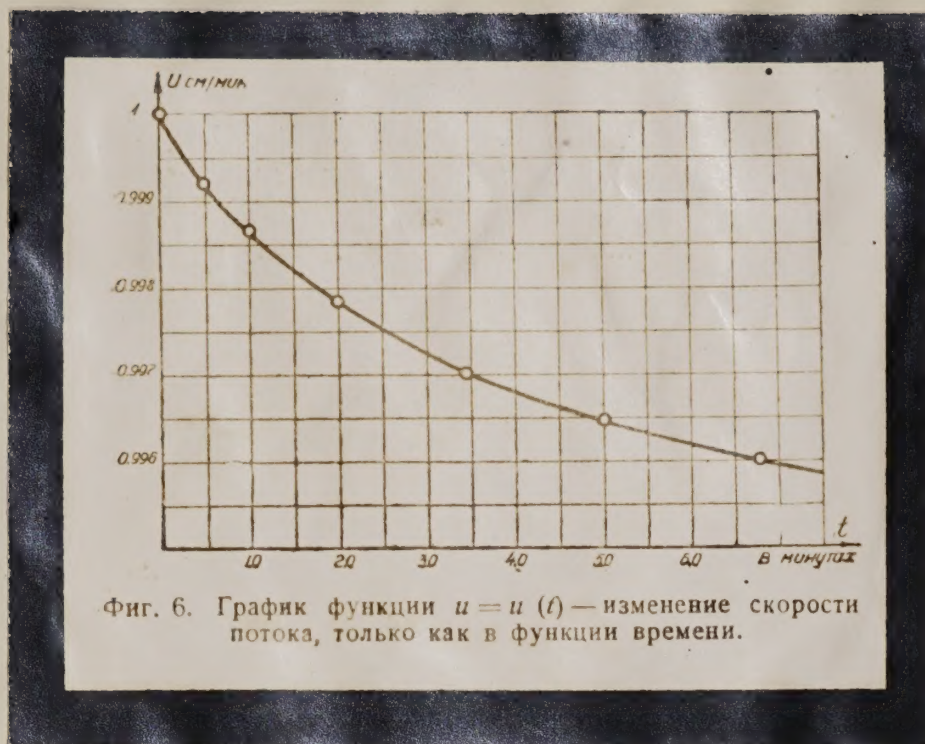


Figure 6.
Graphic of function $u = u(t)$ change of the stream's velocity as function

Given value of function $u = u(t)$ on Fig. 6 answers to the following data:

$$u_0 = 1 \text{ cm/min} \quad d = 0.006 \text{ cm.}$$

$$n_0 = 0.1 \quad \eta_0 = 5$$

29. Equation determining form of function $\eta = \eta(t)$ rewrite eq. (95), obtained as the result of the simultaneous solution of eq. (24) and (29) as follows:

$$+ \frac{2}{n^2 u} \cdot \frac{d\eta}{dt} = 1 \quad (180)$$

Substituting the value of n from eq. (63) and u from eq. (73), we get:

$$+ \frac{2d \cdot c_1^2 \cdot c_2}{n^2 e^{2c_1^2}} \cdot d\eta = dt \quad (181)$$

η^2 Expanding the function $e^{2c_1^2}$ into series of power η and neglecting all members of

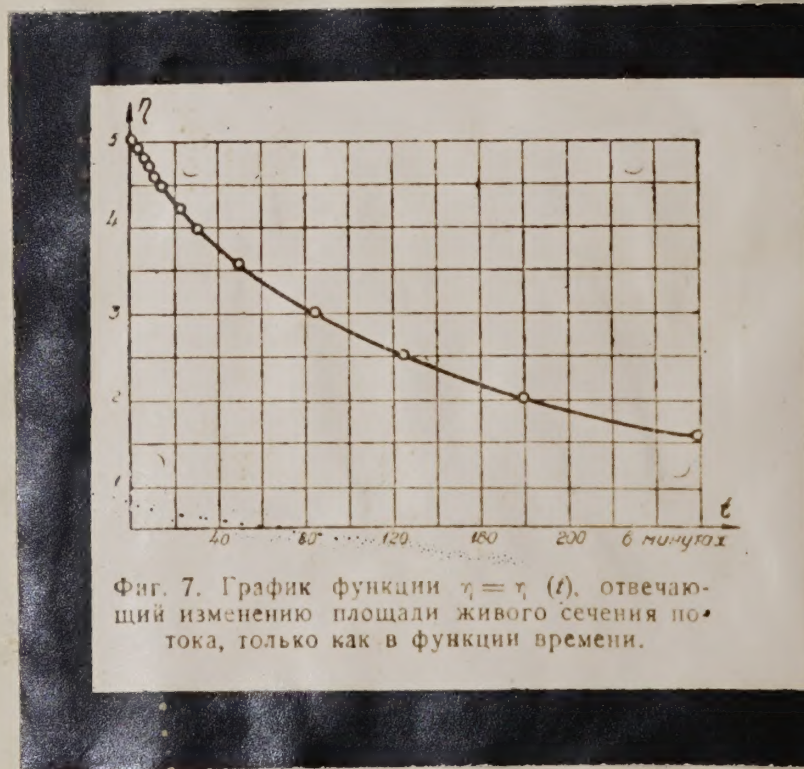


Figure 7.
Graphic of function $\eta = \eta(t)$ answering to the change of cross-section area

Figure 6.
Graph of function $w = w(\lambda)$ change
of the stream's velocity as function

Given value of function
 $w = w(\lambda)$ on fig. 6 answers
to the following data:
 $w_0 = 1 \text{ cm/min}$ $w = 0.001$

$w_0 = 0.1$ $w = 0.001$
23. Equation determining
form of function $w = w(\lambda)$
rewrite eq. (22), obtained as
the result of the simultaneous
solution of eq. (24) and (25)
as follows:

$$+ \frac{2}{\lambda^2} \cdot \frac{d\lambda}{d\lambda} = 1 \quad (26)$$

Substituting the value of w
from eq. (23) and after eq.
(25), we get:

$$+ \frac{2}{\lambda^2} \cdot \frac{d\lambda}{d\lambda} = 0 \quad (27)$$

Expanding the function
into series of power

Figure 7.
Graph of function $w = w(\lambda)$ answer-
ing to the change of cross-section area

the series except first two since series rapidly converges and very small magnitude of the members of it. (See eq. (73) and (170)), we have:

$$2d\epsilon^2\epsilon_2\left(\frac{1}{\eta^2} - \frac{1}{2\epsilon_2^2}\right)d\eta = dt \quad (182)$$

Integral of this equation is:

$$2d\epsilon^2\epsilon_2\left(-\frac{1}{\eta} - \frac{\eta}{2\epsilon_2^2}\right) = t + D_2 \quad (183)$$

Substituting the value of the constant of the integration which is found from eq. (183) by primary conditions, we have:

$$\frac{2d \cdot e^{\frac{\eta_0^2}{2}} \cdot \eta_0}{u_0 \eta_0^2} \left(\frac{\eta_0 - \eta}{\eta}\right) + \frac{d \cdot e^{\frac{\eta_0^2}{2}}}{u_0} (\eta_0 - \eta) = t - t_0 \quad (184)$$

or

$$\frac{2d \cdot e^{\frac{\eta_0^2}{2}} \cdot \eta_0}{u_0 \eta_0^2} \left[\left(\frac{\eta_0 - \eta}{\eta}\right) + \frac{\eta_0^2}{2\eta_0} (\eta_0 - \eta)\right] = t - t_0 \quad (185)$$

For practical purposes since the second member in brackets is very small, it may be recommended to use the following formula:

$$\frac{2d \cdot e^{\frac{\eta_0^2}{2}} \cdot \eta_0}{u_0 \eta_0^2} \left(\frac{\eta_0 - \eta}{\eta}\right) = t - t_0 \quad (186)$$

or, solving eq. (186) with respect to η we have

$$\eta = \frac{\eta_0}{\left[\frac{(t-t_0)u_0\eta_0^2}{2de^{\frac{\eta_0^2}{2}}\eta_0} + 1\right]} \quad (186')$$

30. Consideration of equation (185).

From the eq. (185) it is not difficult to see:

$$1. \text{ At } \eta \rightarrow 0 (t - t_0) \rightarrow \infty \quad (187)$$

Therefore, at any values of η_0 the deposition of the silt was continued in duration of infinitely large interval of time, but the observations and specially performed experiments do not prove it.

Therefore, it is necessary to find the lower limit of the decrease of the that is such limit after approaching of which the solid particles carried by the stream could not penetrate into ground medium because the size of particles must be obviously larger than the pores of ground medium. To this condition, in considered problem, answers such condition of the ground skeleton (due to the deposition of the silt) at which the pores of the latter are completely blocked with solid particles carried by stream.

Therefore, accepting the permeability of such ground skeleton equal to ρ_0 the minimal value of η will be:

$$\eta_{\min} = \frac{\rho_0' d}{d} = \rho_0' \quad (188)$$

the series except that the series rapidly converges and very few
 methods of the numbers of it. (see eq. (17) and (17a), we have:

$$(122) \quad 2\pi C \left(\frac{1}{\pi} - \frac{1}{2C} \right) \phi = 4C$$

Integral of this equation is:

$$(123) \quad 2\pi C \left(\frac{1}{\pi} - \frac{1}{2C} \right) = t + C$$

Substituting the value of the constant of the integration which is found
 from eq. (123) in primary condition, we have:

$$(124) \quad \frac{2\pi C \phi}{\pi} \left(\frac{1}{\pi} - \frac{1}{2C} \right) + \frac{1}{2C} \left(\frac{1}{\pi} - \frac{1}{2C} \right) = C - C$$

$$(125) \quad \frac{2\pi C \phi}{\pi} \left[\left(\frac{1}{\pi} - \frac{1}{2C} \right) + \frac{1}{2C} \left(\frac{1}{\pi} - \frac{1}{2C} \right) \right] = C - C$$

For practical purposes since the second member in brackets is very small,
 it may be recommended to use the following formula:

$$(126) \quad \frac{2\pi C \phi}{\pi} \left(\frac{1}{\pi} - \frac{1}{2C} \right) = C - C$$

or, solving eq. (126) with respect to ϕ we have

$$(127) \quad \phi = \frac{\pi(C - C)}{2\pi C \left(\frac{1}{\pi} - \frac{1}{2C} \right)}$$

50. Consideration of equation (127).

From the eq. (127) it is not difficult to see:

$$(128) \quad \phi \rightarrow 0 \text{ as } C \rightarrow \infty$$

Therefore, at any value of the deposition of the silt was continued
 in duration of relatively large interval of time, but the observation and
 especially previous experiments do not prove it.

Therefore, it is necessary to find the lower limit of the duration of
 the that is such limit after approaching of which the solid particles
 carried by the stream could not penetrate into ground medium because the
 size of particles must be obviously larger than the pores of ground medium.
 To this condition, in considered problem, answers such condition of the
 ground medium (due to the deposition of the silt) at which the pores of
 the latter are completely filled with solid particles carried by stream.

Therefore, according to the necessity of such ground medium equal to
 the minimal value of ϕ will be:

$$(129) \quad \phi_{min} = \frac{C - C}{\pi}$$

with practical respect ρ_0' varies within limits 0.4 to 0.28

Therefore, the limits of the decrease η will be

$$\eta_0 \geq \eta \geq \rho_0' \quad (189)$$

2. Decrease of η in time occurs as more intensively as greater the values of initial velocity of stream and the concentration of it; and as smaller initial cross-section area of percolation tube and size of solid particles carried by stream.

And finally, it is interesting to note that eq. (186) answers to the hyperbolical law of change $\eta = \eta(t)$. This is not difficult to see from the graphical interpretation of eq. (186). (See Figure 7).

On Figure 7 is presented the change $\eta = \eta(t)$ for following data:

$$\eta_0 = 5 \quad d = 0.006$$

$$u_0 = 5 \text{ cm/min} \quad \eta_0 = 0.01$$

31. Equation determining form of function T of period of deposition of silt (complete?). In practical respect, it is very important to know the time in which deposition (complete?) of silt into ground skeleton. Lets call it T period of (complete) deposition of silt. The value for this period may be obtained very easily on the basis of consideration of eq. (185); for this purpose it is sufficient to substitute into eq. (185) the value η of equal to $\eta_{\min}$

Then we have

$$T = \frac{2d e^{\frac{\eta_0^2}{2}} \eta_0}{u_0 \eta_0^2} \left[\left(\frac{\eta_0 - \rho_0'}{\rho_0'} \right) + \frac{\eta_0^2}{2\eta_0} (\eta_0 - \rho_0') \right] \quad (190)$$

or, having in mind the remass about eq. (185)

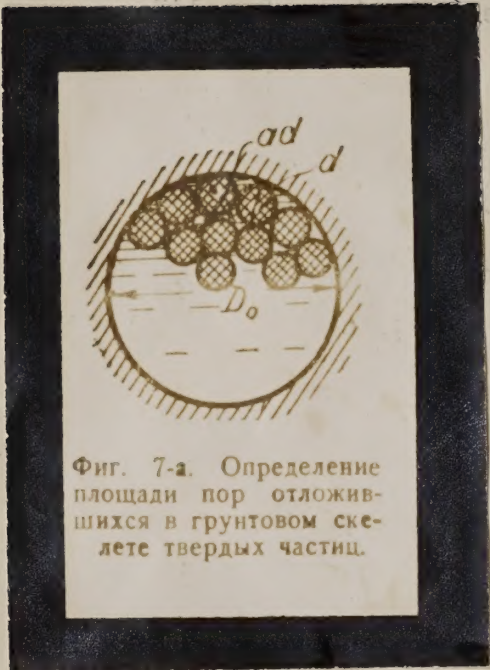
$$T = \frac{2d e^{\frac{\eta_0^2}{2}} \eta_0}{u_0 \eta_0^2} \left(\frac{\eta_0 - \rho_0'}{\rho_0'} \right) \quad (191)$$

Therefore, the magnitude of this period is directly proportional to the square of the diameter of percolation tube and to diameter of solid particles carried by stream and inversely proportional to the initial velocity of movement of the stream and to the square of the percentage of concentration.

32. Equation determining for function $Q = Q(t)$

For determination of the discharge of the stream as function of time according to eq. (60) we have:

$$q(t) = u(t) Q(t) \quad (192)$$



Фиг. 7-а. Определение площади пор отложившихся в грунтовом скелете твердых частиц.

with practical respect ϕ varies within limits 0.4 to 0.58. Therefore, the limits of the decrease ϕ will be

(188)

$$\phi \geq \phi' \geq \phi''$$

2. Decrease of ϕ in time occurs as more intensively as greater the values of initial velocity of stream and the concentration of it; and as smaller initial cross-section area of percolation tube particles carried by stream.

And finally, it is interesting to note that an hyperbolic law of change $\phi = \phi(t)$ This is not difficult to verify graphically (see Figure 7).

On Figure 7 is presented the change $\phi = \phi(t)$

$$\phi = 2 \quad \phi = 0.002$$

$$\phi = 2 \text{ cm/min} \quad \phi = 0.001$$

31. Equation determining form of function $\phi(t)$ (complete). In practical respect, it is known the time in which deposition (complete) of solid particles call it T period of (complete) deposition of solid particles. This period may be obtained very easily on the basis of (185); for this purpose it is sufficient to substitute the values equal to ϕ_{min} .

Then we have

$$T = \frac{2 \phi_{min} \phi_0}{\phi_0 - \phi_{min}} \left[\frac{\phi_0 - \phi_{min}}{\phi_0} + \frac{\phi_{min}}{\phi_0} (\phi_0 - \phi_{min}) \right] \quad (190)$$

or, having in mind the remark about eq. (185)

(191)

$$T = \frac{2 \phi_{min} \phi_0}{\phi_0 - \phi_{min}} \left(\frac{\phi_0 - \phi_{min}}{\phi_0} + \frac{\phi_{min}}{\phi_0} (\phi_0 - \phi_{min}) \right)$$

Therefore, the magnitude of this period is directly proportional to the square of the diameter of percolation tube and to diameter of solid particles carried by stream and inversely proportional to the initial velocity of movement of the stream and to the square of the percentage of concentration.

32. Equation determining for function $\phi = \phi(t)$

For determination of the discharge of the stream as function of time according to eq. (80) we have:

(192)

$$q(t) = u(t) S(t)$$

where Q discharge of the stream through (cross) cross-section of one percolation tube, φ area of cross-section also of one tube.

Now, if deposited in the pores of ground skeleton solid particles, carried by stream, also will be permeable, as for instance in case when the stream is saturated with sandy particles, then the cross-section area of the percolation tube may be determined as follows:

$$Q(t) = \frac{\pi D^2}{4} + w_k \quad (193)$$

where w_k the area of the pores of the deposited in given cross-section of the stream solid particles.

From Figure 7, it is not difficult to see that

$$dw_k = \frac{\rho_0'^2 \pi d^2}{4} \left(\frac{D}{d} - 1 \right) d\eta \quad (194)$$

and therefore

$$w_k = \frac{\pi \rho_0'^2 d^2}{4} \left(\frac{\eta^2}{2} - \eta \right) + C_9 \quad (195)$$

Substituting the value of the constant of the integration which is obtained from eq. (195) by satisfying it with initial conditions, we have:

$$w_k = \frac{\pi \rho_0'^2 d^2}{8} \left[(\eta_0^2 - \eta^2) - 2(\eta_0 - \eta) \right] \quad (196)$$

therefore

$$Q(t) = \frac{\pi d^2}{4} \left\{ \eta^2 + \frac{\rho_0'^2}{2} [(\eta_0^2 - \eta^2) - 2(\eta_0 - \eta)] \right\} \quad (197)$$

If the deposits in the pores of ground medium solid particles have no permeability, or if it is practically negligible, then the area of the percolation tube will be

$$w(t) = \frac{\pi d^2}{4} \cdot \eta^2 \quad (198)$$

But since $u = u(t)$ and $\eta = \eta(t)$ are known functions determined by eq. (176) and (186), then to find the form of $q = q(t)$ is not hard. Solving eq. (176) and (186) with respect u and η and substituting their value into eq. (192), we get the corresponding case of eq. (198):

$$q = q(t) = \frac{\pi d^2}{4} \left[u_0 e^{\frac{\eta_0^2}{2} \left(\frac{(t-t_0) u_0 \eta_0^2}{2 d e^{\frac{\eta_0^2}{2} \eta_0} + 1 \right)^{-1}} \right] \cdot \left[\frac{\eta_0^2}{\left(\frac{(t-t_0) u_0 \eta_0^2}{2 d e^{\frac{\eta_0^2}{2} \eta_0} + 1 \right)^2} \right] \quad (199)$$

and if we denote:

$$K(t) = \frac{(t-t_0) u_0 \eta_0^2}{2 d e^{\frac{\eta_0^2}{2} \eta_0}}$$

where Q discharge of the stream through (cross) cross-section of one per-
colation tube, Q area of cross-section also of one tube.

Now, if deposited in the pores of ground skeleton solid particles,
carried by stream, also will be deposited, as for instance in case when
the stream is saturated with sandy particles, then the cross-section area
of the percolation tube may be determined as follows:

$$(192) \quad Q(t) = \frac{\pi d^2}{4} v + w_t$$

whereas, the area of the pores of the deposited in given cross-section of
the stream solid particles.

From Figure 7, it is not difficult to see that

$$(193) \quad Q_{max} = \frac{\pi d^2 v}{4} (1 - \frac{w}{Q})$$

and therefore

$$(194) \quad w_t = \frac{\pi d^2 v}{4} (1 - \frac{Q}{Q_{max}}) + C$$

Substituting the value of the constant of the integration which is ob-
tained from eq. (194) by satisfying it with initial conditions, we have:

$$(195) \quad w_t = \frac{\pi d^2 v}{4} [1 - (1 - \frac{Q}{Q_{max}})]$$

therefore

$$(196) \quad Q(t) = \frac{\pi d^2 v}{4} \left\{ 1 + \frac{Q}{Q_{max}} \left[1 - (1 - \frac{Q}{Q_{max}}) \right] \right\}$$

If the deposits in the pores of ground medium solid particles have no
permeability, or if it is practically negligible, then the area of the per-
colation tube will be

$$(197) \quad w(t) = \frac{\pi d^2 v}{4} Q$$

But since $w = w(t)$ and $Q = Q(t)$ are known functions determined by eq.
(196) and (197), then to find the form of $Q(t)$ is not hard. Solving eq.
(196) and (197) with respect to w and substituting their value into
eq. (197), we get the corresponding case of eq. (196):

$$(198) \quad Q = \frac{\pi d^2 v}{4} \left[\frac{1 + \frac{Q}{Q_{max}} \left(1 - \frac{Q}{Q_{max}} \right)}{1 + \frac{Q}{Q_{max}} \left(1 - \frac{Q}{Q_{max}} \right)} \right]$$

and if we denote:

$$K(t) = \frac{1 + \frac{Q}{Q_{max}} \left(1 - \frac{Q}{Q_{max}} \right)}{1 + \frac{Q}{Q_{max}} \left(1 - \frac{Q}{Q_{max}} \right)}$$

(199)

then eq. (199) may be rewritten as:

$$q = q(t) = \frac{\pi d^2}{4} u_0 e^{\frac{\pi_0^2}{2} \left[\frac{1}{(k+1)^2} - 1 \right]} \cdot \frac{q_0^2}{(k+1)^2} \quad (201)$$

For the case corresponding to eq. (197), we have:

$$q = \frac{\pi d^2}{4} u_0 \cdot e^{\frac{\pi_0^2}{2} \left[\frac{1}{(k+1)^2} - 1 \right]} \cdot \left\{ \frac{q_0^2}{(k+1)^2} + \frac{P_0'^2}{2} \left[\left(q_0^2 - \frac{q_0^2}{(k+1)^2} \right) - 2 \left(q_0 - \frac{q_0}{k+1} \right) \right] \right\} \quad (202)$$

But practically, we are interested in the change of the discharge of the stream along whole considered region of ground skeleton.

The difficulty of the solution of this question is conditioned by absence of the law of the distribution of the velocities on the area of cross-section of given ground skeleton's region; if the latter is known, then it is not difficult principally for transition from elements of tube to the whole stream.

In case under pressure, in "basic movement", rectilinear movement of the stream as it takes place in considered here problem, with condition that the average velocities of the movement in all percolation tubes are the same, transition from the discharge of flow to the discharge of stream in whole region is realized by elementary integration. And equations for the discharge will be such.

$$1. \quad Q = Q_0 P_0 u_0 e^{\frac{\pi_0^2}{2} \left[\frac{1}{(k+1)^2} - 1 \right]} \cdot \left\{ \frac{1}{(k+1)^2} + \frac{P_0'^2}{2} \left[\left(1 - \frac{1}{(k+1)^2} \right) - 2 \left(\frac{1}{2} - \frac{1}{q_0(k+1)^2} \right) \right] \right\} \quad (203)$$

for the cases, when the solid particles deposited in the pores of ground medium are permeable.

$$2. \quad Q = Q_0 P_0 u_0 \cdot e^{\frac{\pi_0^2}{2} \left[\frac{1}{(k+1)^2} - 1 \right]} \cdot \frac{1}{(k+1)^2} \quad (204)$$

for the cases when they are practically nonpermeable.

For practical purposes, these formulae may be considerably simplified, because usual concentration in nature is not higher than 10%, and therefore, it may be accepted with high degree of accuracy:

$$e^{\frac{\pi_0^2}{2} \left[\frac{1}{(k+1)^2} - 1 \right]} \cong 1 \quad (205)$$

Then for the first case:

$$Q = Q_0 P_0 u_0 \left\{ \frac{1}{(k+1)^2} + \frac{P_0'^2}{2} \left[\left(1 - \frac{1}{(k+1)^2} \right) - 2 \left(\frac{1}{2} - \frac{1}{q_0(k+1)^2} \right) \right] \right\} \quad (206)$$

and for second:

$$Q(t) = Q_0 P_0 u_0 \frac{1}{(k+1)^2} \quad (207)$$

then on. (128) may be rewritten as:

$$(201) \quad \gamma = \frac{1}{2} \left(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} \right) \left[\frac{1}{\gamma_0} - \frac{1}{\gamma_1} \right]$$

For the case corresponding to eq. (127), we have:

$$(202) \quad \gamma = \frac{1}{2} \left(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} \right) \left[\frac{1}{\gamma_0} - \frac{1}{\gamma_1} \right] + \frac{1}{2} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_1} \right) \left[\frac{1}{\gamma_0} - \frac{1}{\gamma_1} \right]$$

But practically, we are interested in the change of the discharge of the stream along whole considered region of ground skeleton.

The difficulty of the solution of this question is conditioned by absence of the law of the distribution of the velocities on the area of cross-section of given ground skeleton's region; if the latter is known, then it is not difficult principally for transition from elements of cube to the whole stream.

In case under pressure, in "static movement", rectilinear movement of the stream as it takes place in considered here problem, with condition that the average velocities of the movement in all permeation tubes are the same, transition from the discharge of flow to the discharge of stream in whole region is realized by elementary integration. And equations for the discharge will be such.

$$(203) \quad Q = \frac{1}{2} \left(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} \right) \left[\frac{1}{\gamma_0} - \frac{1}{\gamma_1} \right] \left[\frac{1}{\gamma_0} - \frac{1}{\gamma_1} \right]$$

For the cases, when the solid particles deposited in the pores of ground medium are impermeable.

$$(204) \quad Q = \frac{1}{2} \left(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} \right) \left[\frac{1}{\gamma_0} - \frac{1}{\gamma_1} \right] \left[\frac{1}{\gamma_0} - \frac{1}{\gamma_1} \right]$$

For the cases when they are practically nonpermeable.

For practical purposes, these formulas may be considerably simplified, because usual concentration in nature is not higher than 10%, and therefore, it may be accepted with high degree of accuracy:

$$(205) \quad \gamma \approx 1$$

Then for the first case:

$$(206) \quad Q = \frac{1}{2} \left(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} \right) \left[\frac{1}{\gamma_0} - \frac{1}{\gamma_1} \right] \left[\frac{1}{\gamma_0} - \frac{1}{\gamma_1} \right]$$

and for second:

$$(207) \quad Q = \frac{1}{2} \left(\frac{1}{\gamma_0} + \frac{1}{\gamma_1} \right) \left[\frac{1}{\gamma_0} - \frac{1}{\gamma_1} \right] \left[\frac{1}{\gamma_0} - \frac{1}{\gamma_1} \right]$$

in all formulae denotes: Q_0 . Geometrical area of cross-section (Normal to direction of the movement) in given region of ground skeleton.
 P_0 initial permeability of ground medium. The minimal discharge, which will pass through ground medium after it "complete" silting up (in practical respect the value is of great importance) will be determined by substitution into eq. (206) and (207) the value of K corresponding to the period of the deposition of the silt,

$$K_{max} = K = \frac{T \cdot u_0 \cdot \eta_0^2}{2 d e^{\frac{\eta_0^2}{2}} \cdot \eta_0} \quad (209)$$

that is

$$Q_{min} = Q_0 P_0 u_0 \left\{ \frac{1}{(K_{max} + 1)^2} + \frac{P_0'}{2} \left[\left(1 - \frac{1}{(K_{max} + 1)^2} \right) - 2 \left(\frac{1}{\eta_0} - \frac{1}{\eta_0 (K_{max} + 1)} \right) \right] \right\} \quad (209)$$

for first case corresponding to eq. (198) and

$$Q_{min} = Q_0 \cdot P_0 \cdot u_0 \frac{1}{(K_{max} + 1)^2} \quad (210)$$

for second case corresponding to eq. (197).

And finally, it is interesting to note that for the case when the deposited into ground medium solid particles (are permeable) have percolating ability and maximal decrease of the discharge of the stream may be expressed as:

$$Q_{min. min} = Q_0 \cdot P_0 \cdot u_0 \cdot \frac{P_0'^2}{2} = \frac{Q_0 \cdot P_0'^2}{2} \quad (211)$$

For instance, if $P_0' = 0.30$ then

$$Q_{min min} = 0.045 \cdot Q_0 \quad (212)$$

Besides, we wish to emphasize once more that P_0' characterizes the diameter of the pore in the deposited into ground skeleton solid particles.

33. Equation determining form of function $\eta = \eta(t)$

Substituting the value of $\frac{\partial \eta}{\partial t}$ into eq. (180) from eq. (62) we have:

$$\frac{2d}{n^2 u} \cdot C_1 \frac{\partial \eta}{\partial t} = 1 \quad (213)$$

and also, remembering that $u = u_0 e^{\frac{\eta^2 - \eta_0^2}{2}}$ we have

$$\frac{2d \eta_0 e^{\frac{\eta_0^2}{2}}}{\eta_0 u_0} \cdot \frac{d\eta}{n^2 e^{\frac{\eta^2}{2}}} = dt \quad (214)$$

Expanding the function η in $e^{\frac{\eta^2}{2}}$ power series and neglecting all members except the first two according to the remarks made in the derivation of eq. (181) we have

$$\frac{2d \eta_0 e^{\frac{\eta_0^2}{2}}}{\eta_0 u_0} \cdot \left[\frac{1}{\eta^2} - \frac{1}{2} \right] d\eta = dt \quad (215)$$

In all formulae denoted by 2. Geometrical area of cross-section (normal to direction of the movement) in given region of ground skeleton. P_0 initial permeability of ground medium. The minimal discharge, which will pass through ground medium after it "completes" sitting up (in practical respect the value is of great importance) will be determined by substitution into eq. (200) and (207) the value of K corresponding to the period of the deposition of the silt.

$$K_{max} = K = \frac{7.46 \cdot 10^{-5}}{2.4 \cdot 10^{-5}} \quad (208)$$

that is

$$Q_{min} = 2.4 \cdot 10^{-5} \left\{ \frac{1}{(K_{max})^2} + \frac{1}{(K_{max})^2} - \frac{1}{(K_{max})^2} - \frac{1}{(K_{max})^2} \right\} \quad (209)$$

for first case corresponding to eq. (198) and

$$Q_{min} = 2.4 \cdot 10^{-5} \cdot \frac{1}{(K_{max} + 1)^2} \quad (210)$$

for second case corresponding to eq. (197).

And finally, it is interesting to note that for the case when the deposited into ground medium solid particles (are permeable) have permeability and maximal decrease of the discharge of the stream may be expressed as:

$$Q_{min} = 2.4 \cdot 10^{-5} \cdot \frac{1}{2} = \frac{1.2 \cdot 10^{-5}}{2} \quad (211)$$

For instance, if $P_0 = 0$ is then

$$Q_{min} = 0.042 \cdot 10^{-5} \quad (212)$$

Besides, we wish to emphasize once more that P_0 characterizes the diameter of the pore in the deposited into ground skeleton solid particles.

3. Equation determining form of function $W = W(t)$

Substituting the value of Q into eq. (160) from eq. (82) we have:

$$1 = \frac{2.4}{W^2} \cdot \frac{1}{6.5} = 1 \quad (213)$$

and also, remembering that $W = W_0 \cdot e^{\frac{at}{2}}$ we have

$$1 = \frac{2.4}{W_0^2} \cdot \frac{1}{6.5} = 1 \quad (214)$$

Expanding the function W in power series and neglecting all members except the first two according to the remarks made in the derivation of eq. (161) we have

$$1 = \frac{2.4}{W_0^2} \cdot \frac{1}{6.5} \left[\frac{1}{W_0^2} - \frac{1}{2} \left(\frac{at}{2} \right)^2 \right] \quad (215)$$

Integral of this equation is:

$$\frac{2d\eta_0 e^{\frac{\eta_0^2}{2}}}{\eta_0 u_0} \left[-\frac{1}{\eta} - \frac{\eta}{2} \right] = t + D_3 \quad (216)$$

Substituting the value of constant of the integration we have

$$\frac{2d\eta_0 e^{\frac{\eta_0^2}{2}}}{u_0 \eta_0^2} \left[\frac{\eta_0 - \eta}{\eta} + \frac{\eta_0(\eta_0 - \eta)}{2} \right] = t - t_0 \quad (217)$$

For practical purposes, it is possible to neglect the second member in the brackets; then eq. (217) rewrites as:

$$\frac{2d\eta_0 e^{\frac{\eta_0^2}{2}}}{u_0 \eta_0^2} \cdot \left(\frac{\eta_0 - \eta}{\eta} \right) = t - t_0 \quad (218)$$

or, solving with respect to η

$$\eta = \frac{\eta_0}{\left[\frac{(t - t_0) u_0 \eta_0^2}{2 d e^{\frac{\eta_0^2}{2}} \eta_0} + 1 \right]} \quad (219)$$

34. Equation determining for the function of $H = H(t)$ rewriting eq. (92) as follows;

$$\frac{dH}{dt} = -\eta \left(1 + \frac{\eta^2}{2} \right) \cdot u \quad (220)$$

Substituting the value of η from eq. (107) and u from eq. (115) we have:

$$\frac{dH}{\eta_0 \cdot \left(e^{-\frac{(H_0 - H)\eta_0}{2 d \eta_0}} \right) \cdot \left(1 + \frac{1}{2} \eta_0^2 e^{\frac{-(H_0 - H)\eta_0}{2 d \eta_0}} \right) u_0 e^{\frac{\eta_0^2}{2} \left(e^{\frac{2(H_0 - H)\eta_0}{\eta_0^2 d}} - 1 \right)}} = -dt \quad (221)$$

But, having in mind eq. (176), we may accept with sufficient accuracy for practical purposes that:

$$u_0 e^{\frac{\eta_0^2}{2} \left(e^{\frac{2(H_0 - H)\eta_0}{\eta_0^2 d}} - 1 \right)} \approx u_0 \quad (222)$$

Besides, since in practical problems is not higher than 0.1, then the member in brackets which is smaller than unity may be neglected. Then eq. (221) takes the following shape:

$$\frac{e^{\frac{(H_0 - H)\eta_0}{2 d \eta_0}} \cdot dH}{\eta_0 u_0} = dt \quad (223)$$

Integral of this equation:

$$\frac{2 d \eta_0}{\eta_0^2 u_0} \cdot e^{\frac{(H_0 - H)\eta_0}{2 d \eta_0}} = t + D_4 \quad (224)$$

or, substituting the value of the integration constant

$$\frac{2 d \eta_0}{\eta_0^2 u_0} \left(e^{\frac{(H_0 - H)\eta_0}{2 d \eta_0}} - 1 \right) = t - t_0$$

Integral of this equation is:

(216)
$$\frac{2 \log \frac{1}{2} e^{\frac{1}{2} \log \frac{1}{2}}}{\log \frac{1}{2}} \left[-\frac{1}{n} - \frac{1}{2} \right] = \frac{1}{2} + \frac{1}{2}$$

Substituting the value of constant of the integration we have

(217)
$$\frac{2 \log \frac{1}{2} e^{\frac{1}{2} \log \frac{1}{2}}}{\log \frac{1}{2}} \left[\frac{1}{n} + \frac{1}{2} \log \frac{1}{2} \right] = \frac{1}{2} - \frac{1}{2}$$

For practical purposes, it is possible to neglect the second member in the brackets; then eq. (217) rewrites as:

(218)
$$\frac{2 \log \frac{1}{2} e^{\frac{1}{2} \log \frac{1}{2}}}{\log \frac{1}{2}} \left(\frac{1}{n} \right) = \frac{1}{2} - \frac{1}{2}$$

or, solving with respect to n

(219)
$$n = \frac{2 \log \frac{1}{2} e^{\frac{1}{2} \log \frac{1}{2}}}{\log \frac{1}{2}} \left(\frac{1}{2} - \frac{1}{2} \right)$$

24. Equation determining for the function of $n = \frac{1}{2}$ rewrites eq. (22) as follows:

(220)
$$\frac{1}{2} = -n \left(1 + \frac{1}{2} \right) \cdot \frac{1}{2}$$

Substituting the value of n from eq. (219) and $\frac{1}{2}$ from eq. (218) we have:

(221)
$$\frac{1}{2} = \frac{2 \log \frac{1}{2} e^{\frac{1}{2} \log \frac{1}{2}}}{\log \frac{1}{2}} \left(\frac{1}{2} - \frac{1}{2} \right) \cdot \frac{1}{2}$$

But, having in mind eq. (219), we may proceed with sufficient accuracy for practical purposes that:

(222)
$$\frac{1}{2} e^{\frac{1}{2} \log \frac{1}{2}} \left(\frac{1}{2} - \frac{1}{2} \right) \approx \frac{1}{2}$$

Besides, since in practical problems it is not higher than 0.1, then the member in brackets which is smaller than unity may be neglected. Then eq.

(221) takes the following shape:

(223)
$$\frac{1}{2} = \frac{2 \log \frac{1}{2} e^{\frac{1}{2} \log \frac{1}{2}}}{\log \frac{1}{2}} \cdot \frac{1}{2}$$

Integral of this equation:

(224)
$$\frac{2 \log \frac{1}{2} e^{\frac{1}{2} \log \frac{1}{2}}}{\log \frac{1}{2}} \cdot \frac{1}{2} = \frac{1}{2} + \frac{1}{2}$$

or, substituting the value of the integration constant

$$\frac{2 \log \frac{1}{2} e^{\frac{1}{2} \log \frac{1}{2}}}{\log \frac{1}{2}} \left(\frac{1}{2} - \frac{1}{2} \right) = \frac{1}{2} - \frac{1}{2}$$

or

$$H_0 - H = \frac{2d\eta_0}{\eta_0} \ln \left[\frac{\eta_0^2 u_0}{2d\eta_0} (t - t_0) + 1 \right] \quad (226)$$

If in practical problem there will be such outer layer conditions which will guarantee the constancy of n in time then integral of eq. (221) will be

$$\frac{H_0 - H}{\eta_0 u_0} = t - t_0 \quad (227)$$

that is the relation between H and t will be linear.

35. Consideration of equation (225) and (226).

At once, we emphasize the physical meaning of the function $H(t)$. Indeed, the change of pressure only as function of time is not the change of the piezometric pressure in any point of the stream where $\frac{dH}{dt}$ may be accepted to be equal to 0; and latter may be accepted only for $s = D$ (because for this value $s \frac{du}{ds} = 0$ - diameter percolation tube). Therefore, the change of the pressure in time for will correspond to accumulation of the losses of the pressure in this portion. The maximal loss of the pressure in this portion will obviously be equal to the loss of the pressure corresponding to complete blocking of the ground skeleton with the silt, and may be determined from eq. (226) by substitution into latter the value $t - t_0 = T$ period complete blockage, which determined by eq. (191) that is

$$(H_0 - H)_{\max} = \frac{2d\eta_0}{\eta_0} \ln \left[\frac{\eta_0^2 u_0}{2d\eta_0} T + 1 \right] \quad (228)$$

Now, turning eq. (225) we note that the change of the pressure in time occurs as more intensively as larger values have u_0, η_0 and as smaller η_0 (see coeff. at $t - t_0$); besides, the change of the pressure only as function of time in all points of the stream occurs by similar method, that is, if we draw the graphic of function $H = H(t)$ for different points of the stream, then curves $H = H(t)$ will be parallel between themselves.

And finally, we give graphical interpretation of eq. (226). (See Fig. 8)

The given graphic of function $H = H(t)$ corresponding to following initial data

$$\eta_0 = 0.01 \quad u_0 = 6.0 \text{ cm/min}$$

$$\eta_0 = 5.0 \quad H_0 = 4.0 \text{ cm}$$

$$d = 0.006 \text{ cm.}$$

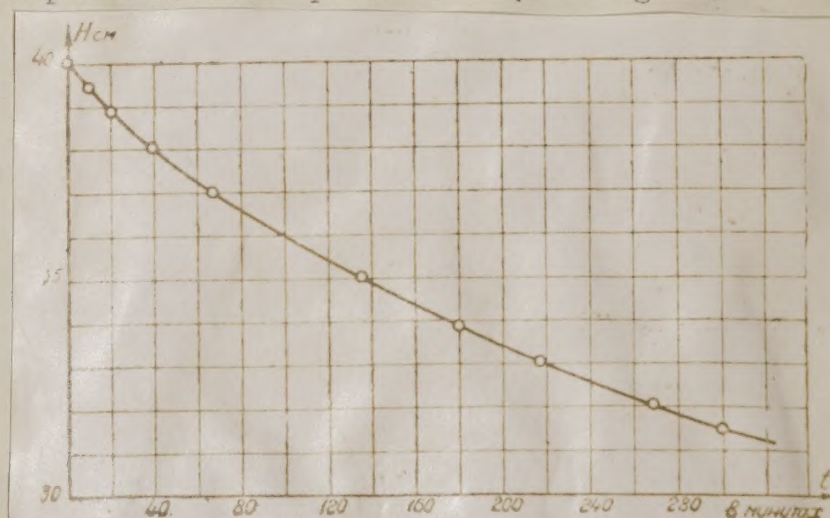
36. Equation determining form of function $u = u(s)$ (as function of transmitted distance)

Rewriting eq. (122) as follows:

$$2d\eta \frac{d\eta}{ds} = n \cdot p \quad (229)$$

$$\text{remembering that } p = n \cdot \eta \quad (230)$$

$$\text{then } \frac{2d}{n^2} \cdot \frac{d\eta}{ds} = 1 \quad (231)$$



Фиг. 8. График функции $H = H(t)$. — Изменение напора в любой точке потока, только как в функции времени.

Figure 8.
Graphic of function $H = H(t)$
The change of the pressure in any point of the stream only as function of time.

$$(226) \quad H_0 - H = \frac{2.4 M_0}{M_0} \ln \left[\frac{H_0}{H} (1 + \frac{H_0}{H}) \right]$$

If in practical problem there will be such outer layer conditions which will guarantee the constancy of n in time then interval of eq. (221) will be

$$(227) \quad \frac{H_0 - H}{M_0} = C - C_0$$

that is the relation between H and t will be linear.

22. Consideration of equation (225) and (226).

At once, we emphasize the physical meaning of the function $H(t)$. Indeed, the change of pressure only as function of time is not the change of the piezometric pressure in any point of the stream where H may be accepted to be equal to 0; and latter may be accepted only for $t=0$ (because for this value $H=0$). Therefore, the change of the pressure in time for will correspond to accumulation of the losses of the pressure in this portion. The maximal loss of the pressure in this portion will obviously be equal to the loss of the pressure corresponding to complete blocking of the ground skeleton with the slit, and may be determined from eq. (226) by substitution into latter the value $t=t_0$ period complete blocking, which determined by eq. (221) that is

$$(228) \quad (H_0 - H)_{max} = \frac{2.4 M_0}{M_0} \ln \left[\frac{H_0}{H_0} (1 + \frac{H_0}{H_0}) \right]$$

Now, turning eq. (225) we note that the change of the pressure in time occurs as more intensively as larger values have M_0 and as smaller M_0 (see eq. 22). Besides, the change of the pressure only as function of time in all points of the stream occurs by similar method, that is, if we draw the graphic of function $H = H(t)$ for different points of the stream, then curves $H = H(t)$ will be parallel between themselves.

And finally, we give graphical interpretation of eq. (226). (see Fig. 8)

The given graphic of function $H = H(t)$ corresponding to following initial data

$$M_0 = 0.01 \quad M_0 = 0.05 \quad M_0 = 0.1 \quad M_0 = 0.2 \quad M_0 = 0.5 \quad M_0 = 1.0$$

23. Equation determining form of function $H = H(t)$ (as function of transmitted distance)

Deriving eq. (122) as follows:

$$(229) \quad \frac{dH}{dt} = -M \cdot q$$

remembering that $q = n \cdot p$

$$(230) \quad \frac{dH}{dt} = -M \cdot n \cdot p$$

$$(231) \quad \frac{dH}{dt} = -M \cdot n \cdot p$$

and having in mind eq. (118) we have:

$$-\frac{2d\zeta_6}{n^4} \cdot \frac{dn}{ds} = 1$$

232

~~and having in mind eq. (118) we have:~~

$$-\frac{2d\zeta_6}{n^4} \cdot dn = ds$$

~~(232)~~

or

(233)

and finally, substitution from eq. (131) and (132) we have:

$$-\frac{2d\zeta_6 \cdot \zeta_9^{\frac{3}{4}}}{4u^{\frac{7}{4}}} du = ds \quad (234)$$

Integral of this equation is

$$\frac{2d\eta_{00}}{3\eta_{00}^2} \cdot \left(\frac{u_{00}}{u}\right)^{\frac{3}{4}} = s + D_4 \quad (235)$$

Substituting the value of the integration constant, we have:

$$\frac{2d\eta_{00}}{3\eta_{00}^2} \left[\left(\frac{u_{00}}{u}\right)^{\frac{3}{4}} - 1 \right] = s - s_0 \quad (236)$$

or solving this equation with respect to u :

$$u = \frac{u_{00}}{\left[\frac{3(s-s_0)\eta_{00}^2}{2d\eta_{00}} + 1 \right]^{\frac{4}{3}}} \quad (237)$$

37. Consideration of equation (236) and (237), we note the following:

$$1. \text{ At } u \rightarrow 0 \quad s - s_0 \rightarrow \infty \quad (238)$$

But such interpretation of obtained equation is not quite right, because obtained equation corresponds only to such portion of the road of the movement of the stream where $\frac{d\eta}{ds} \neq 0$; therefore, minimal value of u will correspond to η_0 on approaching of which the velocity will remain constant along further portion of the movement.

Therefore, the limits of the change u as only function of the will be

$$u_{00} \geq u \geq u_{\eta_0} \quad (239)$$

Concerning the determination of u_{η_0} , its value is very easy to find from eq. (237), for it is sufficient to substitute the value of $s-s_0$ corresponding to η_0 determined from eq. (244).

and having in mind eq. (118) we have:

$$1 = \frac{2b}{a} \cdot \frac{2b}{a} = 1$$

and having in mind eq. (118) we have:

$$- \frac{2b}{a} \cdot \frac{2b}{a} = 1$$

and finally, substitution from eq. (121) and (122) we have:

$$- \frac{2b}{a} \cdot \frac{2b}{a} = 1$$

Integral of this equation is

$$\frac{2b}{a} \cdot \frac{2b}{a} = 1$$

Substituting the value of the integration constant, we have:

$$\frac{2b}{a} \cdot \frac{2b}{a} = 1$$

or solving this equation with respect to x :

$$x = \frac{2b}{a} \cdot \frac{2b}{a} = 1$$

37. Consideration of equation (236) and (237), we note the following:

1. At

$$x \rightarrow 0 \rightarrow 2 \rightarrow \infty$$

But such interpretation of obtained equation is not quite right, because obtained equation corresponds only to such portion of the road of the movement of the stream where $x \rightarrow 0$; therefore, minimal value of x will correspond to $x \rightarrow 0$ on approaching of which the velocity will remain constant along further portion of the movement.

Therefore, the limits of the change x as only function of the will

$$x \rightarrow 0 \rightarrow 2 \rightarrow \infty$$

Concerning the determination of x , its value is very easy to find from eq. (237), for it is sufficient to substitute the value of x corresponding to x determined from eq. (241).

2. The change velocity as $f(s)$, occurs as more intensively as greater value of the concentration of the stream as smaller the diameter of the pores of ground medium. And finally, we give graphical interpretation of eq. (236). (See Fig. 9)

38. Equation determining form of function $\eta = \eta(s)$. Substituting the value of n from eq. (119) into eq. (231), we have:

$$\frac{2d}{c^2} \cdot \eta^2 d\eta = ds \quad (240)$$

Integral of this equation is

$$\frac{2d}{3\eta_{00}^2 \eta_{00}^2} \cdot \eta^3 = s + D_s \quad (241)$$

Substituting the value for constant of integration:

$$\frac{2d}{3\eta_{00}^2 \eta_{00}^2} (\eta^3 - \eta_{00}^3) = s - s_0 \quad (242)$$

or

$$\frac{2d\eta_{00}}{3\eta_{00}^2} \left(\frac{\eta^3}{\eta_{00}^3} - 1 \right) = s - s_0 \quad (243)$$

or solving for η we have:

$$\eta = \eta_{00} \left[\frac{3(s-s_0)\eta_{00}^2}{2d\eta_{00}} + 1 \right]^{\frac{1}{3}} \quad (244)$$

39. Consideration of equations (243) and (244).

1. Obtained equation is correct only in relation to the portion where

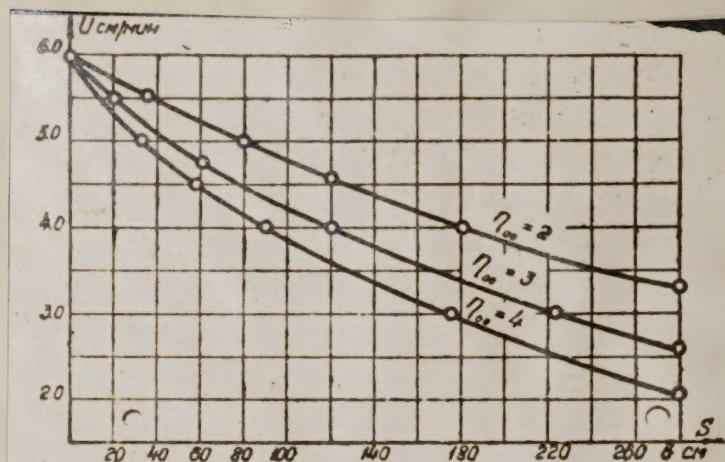
2. The limits of the change η as $f(s)$ for each separate moment of time are

$$\eta_{00} \leq \eta \leq \eta_0 \quad (245)$$

3. The change of η as $f(s)$ occurs as more intensively as greater the value of the concentration of the stream and as lesser the diameter of the pores of the ground medium.

4. From eq. (243) it is easy to see that the law of the deposition of the solid particles along the length of the stream in the process of its movement, indeed, from eq. (63), we have

$$\frac{\eta_{00}}{\eta_{00}} = \frac{\eta_0}{\eta_0} \quad (246)$$



Фиг. 9. График функции $u = u(s)$ для различных моментов времени.

Figure 9.

The graphic of function $u = u(s)$ for different moments of time.

3. The change velocity as $f(z)$, occurs as more intensively as greater value of the concentration of the stream and as lesser the diameter of pores of ground medium. And finally, we give graphical illustration of eq. (232). (See Fig. 3)

35. Substitution determining form of function $\varphi = \varphi(z)$. Substituting the value of n from eq. (212) into eq. (231), we have:

$$\frac{d\varphi}{dz} = \varphi^2 \quad (240)$$

Integral of this equation is

$$\frac{1}{\varphi} = z + C_1 \quad (241)$$

Substituting the value for constant of integration:

$$\frac{1}{\varphi} = z - \frac{1}{\varphi_0} \quad (242)$$

$$\frac{1}{\varphi} = z - \frac{1}{\varphi_0} \quad (243)$$

or solving for φ we have:

$$\varphi = \frac{\varphi_0}{1 - \varphi_0(z - \frac{1}{\varphi_0})} \quad (244)$$

36. Consideration of equations (242) and (244).

1. Obtained equation is correct only in relation to the portion where

2. The limits of the change φ as $f(z)$ for each concrete moment of time are

$$\varphi_0 \leq \varphi \leq \varphi_1 \quad (245)$$

3. The change of φ as $f(z)$ occurs as more intensively as greater the value of the concentration of the stream and as lesser the diameter of the pores of the ground medium.

4. From eq. (242) it is easy to see that the law of the deposition of the solid particles along the length of the stream in the process of its movement, indeed, from eq. (23), we have

$$\frac{d\varphi}{dz} = \varphi^2 \quad (246)$$

Therefore, eq. (243) may be rewritten as follows:

$$\frac{2d\eta_0}{3\eta_0} \left(\frac{\eta^3}{\eta_0^3} - 1 \right) = (s - s_0) M_{00} \quad (247)$$

From this follows that at the same relation of $\frac{\eta}{\eta_0} (s - s_0)$ will increase in time inversely proportional to decrease of M_{00} . Therefore, the same value of η in time along the length of the stream, equals

$$S_{\eta} = \frac{S_1 M_{00}}{M_{00_2}} \quad (248)$$

where S_1 and M_{00} answer to any (some) given moment of time. The same law is easy to see from the graphic 10 (See Figure 10); performed there the change of function $\eta = \eta(s)$ corresponds to data: $\eta_0 = 5, M_0 = 0.01, d = 0.006 \text{ cm}$ for three moments of time corresponding to $\eta_{00} = 4.0, \eta_{00} = 3.00, \eta_{00} = 2.00$

5. And finally note that function η may be immediately expressed as function of two variables t and s :

$$\eta(s, t) = \frac{\eta_0 \left[\frac{3(s-s_0)M_0^2}{2d\eta_0} \frac{1}{\left[\frac{(t-t_0)u_0 M_0^2}{2d\eta_0 e^{\frac{\eta_0^2}{2}}} + 1 \right]} + 1 \right]^{\frac{1}{3}}}{\left[\frac{(t-t_0)u_0 M_0^2}{2d e^{\frac{\eta_0^2}{2}}} + 1 \right]} \quad (249)$$

or, denoting

$$\frac{3(s-s_0)M_0^2}{2d\eta_0} = N \quad (250)$$

and having in mind eq. (200), we have

$$\eta(s, t) = \frac{\eta_0 \left[\frac{N}{K+1} + 1 \right]^{\frac{1}{3}}}{K+1} \quad (251)$$

40. Equation, determining form of function $Q = Q(s)$

For determination of the value of the discharge of the stream as function of s , we use eq. (60) which may be written so:

$$Q(s) = u(s) \cdot \varphi(s) \quad (252)$$

where Q = discharge of the stream through cross-section of one percolation tube, φ = area of the cross-section also of one tube. If deposited into the pores



Figure 10
Graphic of function $\eta = \eta(s)$ for different moments of time.

of ground skeleton solid particles are permeable as for instance in one case, when the stream is saturated with sandy particles, then the area of percolation tube will be determined as follows: (See 32)

$$Q(s) = \frac{\pi d^2}{4} \left\{ \eta^2 + \frac{P_0'}{2} \left[(\eta_0^2 - \eta^2) - 2(\eta_0 - \eta) \right] \right\} \quad (253)$$

And if these particles are not permeable or practically its permeability must be neglected then

$$Q = \frac{\pi d^2}{4} \eta^2 \quad (254)$$

But since $u(s)$ and $\eta(s)$ are known function, which are determined by eq. (237) and (244), then the value of $g = g(s)$ is very easy to find. For this, it is sufficient to substitute this value into eq. (252) and for the case corresponding to eq. (253), we have

$$g = g(s) = \frac{\pi d^2}{4} \left[\frac{u_{00}}{\left[\frac{3(s-s_0) \eta_{00}^2}{2d\eta_{00}} + 1 \right]^{\frac{4}{3}}} \right] \cdot \left\{ \eta_{00}^2 \left[\frac{3(s-s_0) \eta_{00}^2}{2d\eta_{00}} + 1 \right]^{\frac{2}{3}} + \right. \\ \left. + \frac{P_0'}{2} \left\{ \left(\eta_0^2 - \eta_{00}^2 \left[\frac{3(s-s_0) \eta_{00}^2}{2d\eta_{00}} + 1 \right]^{\frac{2}{3}} \right) - 2 \left(\eta_0 - \eta_{00} \left[\frac{3(s-s_0) \eta_{00}^2}{2d\eta_{00}} + 1 \right]^{\frac{1}{3}} \right) \right\} \right\} \quad (255)$$

If we denote

$$M = \frac{3(s-s_0) \eta_{00}^2}{2d\eta_{00}} \quad (256)$$

then eq. (255) will appear as

$$Q(s) = \frac{\pi d^2}{4} \frac{u_{00}}{(M+1)^{\frac{4}{3}}} \cdot \left\{ \eta_{00}^2 (M+1)^{\frac{2}{3}} + \frac{P_0'}{2} \left[\left(\eta_0^2 - \eta_{00}^2 (M+1)^{\frac{2}{3}} \right) - 2 \left(\eta_0 - (M+1)^{\frac{1}{3}} \eta_{00} \right) \right] \right\} \quad (257)$$

For the case corresponding to eq. (254), we have:

$$Q(s) = \frac{\pi d^2}{4} \frac{u_{00} \eta_{00}^2}{(M+1)^{\frac{2}{3}}} \quad (258)$$

For the determination of the discharge of the stream throughout whole region of ground skeleton in considered case, it is sufficient to integrate eq. (257) and (258) through whole area of the cross-section of it. Doing the same as in 32, we get the following expression for the discharges:

In the first case:

$$Q = Q_0 P_0 \frac{u_{00}}{(M+1)^{\frac{4}{3}}} \left\{ (M+1)^{\frac{2}{3}} + \frac{P_0'}{2} \left[\left(1 - (M+1)^{\frac{2}{3}} \right) - 2 \left(\frac{1}{\eta_0} - (M+1)^{\frac{1}{3}} \right) \right] \right\} \quad (259)$$

and for second case: Corresponding to eq. (254)

$$Q(s) = Q_0 P_0 \frac{u_{00} \eta_{00}^2}{(M+1)^{\frac{2}{3}} \eta_0^2} \quad (260)$$

It is easy to see here that the discharge along the length of the stream also decreases, which is very interesting since the area of cross-section along the length of the stream increases: Obviously the decrease of the discharge conditioned by very intensive change of the velocity of the stream along its length.

41. Equation determining form of function $\eta = \eta(s)$ rewriting eq. (232) as follows:

$$\frac{2dC_6}{\eta_4} d\eta = ds \quad (261)$$

Integral of this equation is

$$\frac{2d\eta_{00} \eta_{00}}{3\eta^3} = s + D_7 \quad (262)$$

Substituting value of the constant of the integration:

$$\frac{2d\eta_{00}}{3\eta_{00}} \left(\frac{\eta_{00}^3}{\eta^3} - 1 \right) = s - s_0 \quad (263)$$

or solving eq. (263) for η :

$$\eta = \frac{\eta_{00}}{\left[\frac{3(s-s_0)\eta_{00}^2}{2d\eta_{00}} + 1 \right]^{\frac{1}{3}}} \quad (264)$$

42. Equation determining form of function $H = H(s)$ rewrite eq. (156) as

$$+ \frac{\partial H}{\partial s} = - \left(\frac{H}{2u} + \frac{u}{g} \right) \frac{\partial u}{\partial s} + \frac{\beta u^{\frac{3}{2}}}{u_{00}^{\frac{1}{2}} \eta_{00}^2} \quad (265)$$

From this, it is not difficult to see that the second number in brackets can be neglected without affecting practical accuracy, besides it should be noted that $u = u(s)$ changes relatively on small distance (See eq. 237). Therefore, for s corresponding $\frac{\partial u}{\partial s}$ pressure has considerable value (magnitude). Then eq. (265) may be rewritten as:

$$+ \frac{\partial H}{\partial s} = - \frac{H}{2u} \frac{\partial u}{\partial s} + \frac{\beta u^{\frac{3}{2}}}{u_{00}^{\frac{1}{2}} \eta_{00}^2} \quad (266)$$

or having in mind that $\frac{\partial u}{\partial s}$ has a negative sign

$$\frac{\partial H}{\partial s} = \frac{H}{2u} \cdot \frac{\partial u}{\partial s} + \frac{\beta u^{\frac{3}{2}}}{u_{00}^{\frac{1}{2}} \eta_{00}^2} \quad (267)$$

Multiplying both parts of eq. (267) on Δs

$$+ \Delta H = \frac{H}{2u} \cdot \Delta u + \frac{\beta u^{\frac{3}{2}} \Delta s}{u_{00}^{\frac{1}{2}} \eta_{00}^2} \quad (268)$$

From this it is easy to see that accumulation of the pressure in portion of the movement of the stream Δs composed of two parts:

$$\Delta H_1 = \frac{H}{2u} \cdot \Delta u \quad (269)$$

corresponding to the accumulation of the pressure conditioned by the change of velocity of the stream along its length:

$$\Delta H_2 = \frac{\beta u^{\frac{3}{2}} \Delta s}{u_{00}^{\frac{1}{2}} \eta_{00}^2} \quad (270)$$

It is easy to see here that the discharge along the length of the stream also decreases, which is very interesting since the rate of cross-section along the length of the stream increases. Obviously the decrease of the discharge is conditioned by very intensive change of the velocity of the stream along its length.

41. Equation determining form of function $W = W(x)$ writing eq. (332) as follows:

(341)
$$\frac{dW}{dx} = W$$

Integral of this equation is

(342)
$$\ln W = x + C$$

Substituting value of the constant of the integration:

(343)
$$\ln W = x - 2$$

or solving eq. (343) for W :

(344)
$$W = e^{x-2}$$

42. Equation determining form of function $W = W(x)$ writing eq. (344) as

(345)
$$\frac{dW}{dx} = -\left(\frac{W}{2x} + \frac{W}{2} + \frac{W}{2x^2}\right)$$

From this, it is not difficult to see that the second member of the equation can be neglected without affecting practical accuracy, besides, it should be noted that W changes rapidly on small distance (see eq. 344). Therefore, for a corresponding W , pressure has considerable value (negative). Then eq. (345) can be written as:

(346)
$$\frac{dW}{dx} = -\frac{W}{2x} + \frac{W}{2}$$

or having in mind that $\frac{dW}{dx}$ has a negative sign

(347)
$$\frac{dW}{dx} = \frac{W}{2x} - \frac{W}{2}$$

Multiplying both parts of eq. (347) on dx :

(348)
$$dW = \left(\frac{W}{2x} - \frac{W}{2}\right) dx$$

From this it is easy to see that accumulation of the pressure in portion of the movement of the stream, composed of two parts:

(349)
$$\Delta W = \frac{W}{2x} \Delta x - \frac{W}{2} \Delta x$$

corresponding to the accumulation of the pressure conditioned by the change of velocity of the stream along its length:

(350)
$$\Delta W = \frac{W}{2x} \Delta x$$

corresponding to the accumulation of the pressure conditioned by the presence of the resistance forces to the movement of the stream.

Since, the value of the function $u=u(s)$ is known and is determined by eq. (237), then the change of the pressure due to the presence of the resistances to the movement may be found very easy; that is, substituting the value u from eq. (237):

$$\Delta H_2 = \frac{\beta u_{00}}{\eta_{00}^2} \cdot \frac{\Delta s}{\left[\frac{3(s-s_0)\eta_{00}^2 + 1}{2d\eta_{00}} \right]^2} \quad (271)$$

Integral of this:

$$H_2 = - \frac{\beta_1 u_{00}}{\eta_{00}^2} \cdot \frac{1}{[3(s-s_0)\eta_{00}^2 + 2d\eta_{00}]} + P_8 \quad (272)$$

Substituting value for

$$H_{00} - H_2 = \frac{\beta_1 u_0}{\eta_{00}^2} \left[\frac{1}{2d\eta_{00}} - \frac{1}{[3(s-s_0)\eta_{00}^2 + 2d\eta_{00}]} \right] \quad (273)$$

or,

$$H_{00} - H_2 = \frac{\beta_1 u_{00}}{\eta_{00}^2} \cdot \frac{3(s-s_0) \cdot \eta_{00}^2}{2d\eta_{00} [3(s-s_0)\eta_{00}^2 + 2d\eta_{00}]} \quad (274)$$

or,

$$H_{00} - H_2 = \frac{3\beta_1 u_{00}}{2d\eta_{00}} \cdot \frac{s-s_0}{[3(s-s_0)\eta_{00}^2 + 2d\eta_{00}]} \quad (275)$$

For determinations of the $H_1(s)$ we will do as follows: by integration of eq. (269), we have

$$\frac{H_1}{H_{00}} = \left(\frac{u}{u_{00}} \right)^{\frac{1}{2}} \quad (276)$$

therefore,

$$H_1 = H_{00} \left(\frac{u}{u_{00}} \right)^{\frac{1}{2}} \quad (277)$$

Substituting the value u from eq. (237), we have:

$$\frac{H_1}{H_{00}} = \frac{1}{\left[\frac{3(s-s_0)\eta_{00}^2 + 1}{2d\eta_{00}} \right]^{\frac{2}{3}}} \quad (278)$$

from it:

$$H_{00} - H_1 = H_{00} \left[1 - \frac{1}{\left[\frac{3(s-s_0)\eta_{00}^2 + 1}{2d\eta_{00}} \right]^{\frac{2}{3}}} \right] \quad (279)$$

or,

$$H_{00} - H_1 = H_{00} \left[1 - \frac{(2d\eta_{00})^{\frac{2}{3}}}{[3(s-s_0)\eta_{00}^2 + 2d\eta_{00}]^{\frac{2}{3}}} \right] \quad (280)$$

or,

$$H_{00} - H_1 = H_{00} \left[\frac{[3(s-s_0)\eta_{00}^2 + 2d\eta_{00}]^{\frac{2}{3}} - (2d\eta_{00})^{\frac{2}{3}}}{[3(s-s_0)\eta_{00}^2 + 2\eta_{00}]^{\frac{2}{3}}} \right] \quad (281)$$

Therefore, having in mind eq. (268), we have

$$H_{00} - H = H_{00} \left[1 - \frac{(2d\eta_{00})^{\frac{2}{3}}}{[\eta_{00}^2 \cdot 3(s-s_0) + 2d\eta_{00}]^{\frac{2}{3}}} \right] + \frac{3\beta \cdot \eta_{00}}{2d\eta_{00}} \cdot \frac{s-s_0}{[3(s-s_0)\eta_{00}^2 + 2d\eta_{00}]} \quad (282)$$

or,

$$H_{00} - H = H_{00} \left[1 - \frac{1}{\left[\frac{3(s-s_0)\eta_{00}^2}{2d\eta_{00}} + 1 \right]^{\frac{2}{3}}} \right] + \frac{\eta_{00}(s-s_0)}{\left[\frac{3(s-s_0)\eta_{00}^2}{2d\eta_{00}} + 1 \right] \eta_{00}^2} \cdot \frac{3\beta}{4d^2} \quad (283)$$

For the satisfaction of the eq. (283) to final conditions, which is a such that for $s-s_0=s^0$ to the length of ground region, which is undergo of deposition of silt the $H_{00} - H = H_{00}'$ to actual (working) pressure in the process of the deposition of silt, it is necessary in eq. (283) to substitute instead of $\frac{3\beta}{4d^2}$ it following value

$$\frac{3\beta}{4d^2} = \frac{H_{00} \cdot \eta_{00}^2}{\eta_{00} s^0} \quad (284)$$

Substituting this into eq. (283), we finally get:

$$H_{00} - H = H_{00} \left[1 - \frac{1}{\left[\frac{3(s-s_0)\eta_{00}^2}{2d\eta_{00}} + 1 \right]^{\frac{2}{3}}} \right] + \frac{H_{00}' \cdot \eta_{00}^2 \cdot \eta_{00}(s-s_0)}{\eta_{00} \eta_{00}^2 s^0 \left[\frac{3(s-s_0)\eta_{00}^2}{2d\eta_{00}} + 1 \right]} \quad (285)$$

or having in mind eq. (266)

$$H_{00} - H = H_{00} \left[\left(1 - \frac{1}{(M+1)^{\frac{2}{3}}} \right) + \frac{\eta_{00}^2 \eta_{00}(s-s_{00})}{\eta_{00} \eta_{00}^2 s^0 (M+1)} \right] \quad (285')$$

43. Consideration of eq. (285).

1. At $\eta_{00}=0$ that is, when the Blocking with silt of the ground skeleton does not occur:

$$\frac{H_0 - H}{s - s_0} = \frac{H_0}{s_0} \quad (285')$$

what is accurately corresponding to uniform stream-line movement of the stream.

what is accurately corresponding to uniform stream-line movement of the stream.

(285)

$$\frac{H_0 - H}{z} = \frac{H_0}{z}$$

does not occur:

1. At $H_0 = 0$ when the blocking with slip of the ground skeleton

43. Consideration of eq. (285).

$$H_0 - H = H_0 \left[1 - \frac{1}{(H_0)^2} + \frac{1}{2} \frac{H_0^2 (1 - 2)}{H_0^2} \right] \quad (285)$$

or having in mind eq. (283)

$$H_0 - H = H_0 \left[1 - \frac{1}{2} \frac{H_0^2 (1 - 2)}{H_0^2} + \frac{1}{2} \frac{H_0^2 (1 - 2)}{H_0^2} \right] \quad (286)$$

Substituting this into eq. (283), we finally get:

$$\frac{H_0}{H_0} = \frac{H_0}{H_0}$$

(284)

of $\frac{H_0}{H_0}$ it follows that

of the deposition of slip, it is necessary in eq. (282) to substitute instead of the $H_0 - H$ to actual (working) pressure in the process position of slip the $H_0 - H_0$. Such a substitution is made in eq. (282) to obtain the relation of the length of ground reaction, which is subject to de-

$$H_0 - H = H_0 \left[1 - \frac{1}{2} \frac{H_0^2 (1 - 2)}{H_0^2} + \frac{1}{2} \frac{H_0^2 (1 - 2)}{H_0^2} \right] \quad (287)$$

$$H_0 - H = H_0 \left[1 - \frac{1}{2} \frac{H_0^2 (1 - 2)}{H_0^2} + \frac{1}{2} \frac{H_0^2 (1 - 2)}{H_0^2} \right] \quad (288)$$

Therefore, having in mind eq. (283), we have

$$H_0 - H = H_0 \left[\frac{H_0^2 (1 - 2)}{H_0^2} + \frac{1}{2} \frac{H_0^2 (1 - 2)}{H_0^2} \right] \quad (289)$$

$$H_0 - H = H_0 \left[1 - \frac{1}{2} \frac{H_0^2 (1 - 2)}{H_0^2} \right] \quad (290)$$

2. The change of pressure as $f(s)$ occurs as more intensively as greater the values of the concentration and velocity of the movement of the stream and as smaller the diameter of the pores of the ground skeleton.

3. Obtained equation is obviously justifiable only in the portion where

$$\frac{d\eta}{ds} \neq 0$$

We give the graphical interpretation of obtained eq. (see Fig. 11).

Given on Figure 11, curves $H = H(s)$ correspond to the following data:

$$H^0 = 1 \text{ m}, S^0 = 3.0 \text{ m}, \eta_0 = 5.0$$

$$M_0 = 0.01, d = 0.006 \text{ cm.}$$

and they correspond to the four different moments of time:

$$\eta_{00} = 5, \eta_{00} = 4, \eta_{00} = 3, \eta_{00} = 2$$

respectively. Therefore in the present paper, all function of hydrodynamical elements of the stream are given in final form,

*(Practical application of obtained solution will be given in Chapter III, V, VI, and VII, in which the

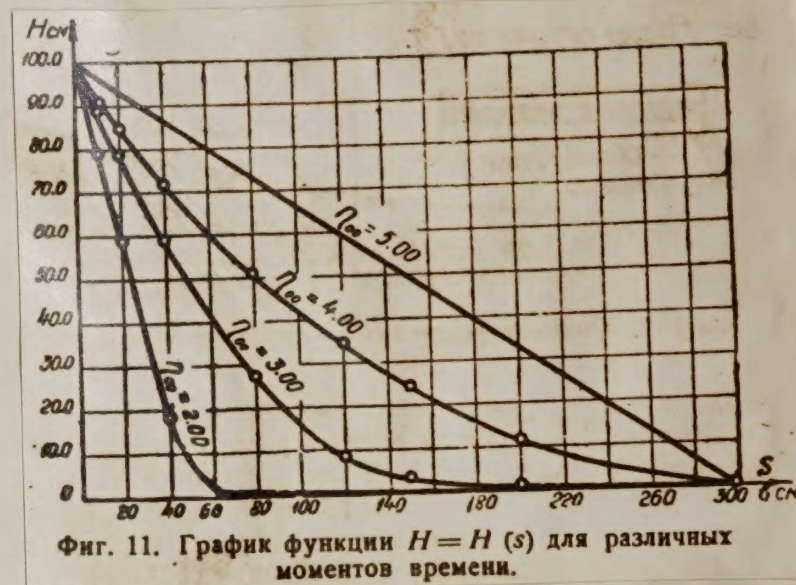
following questions are considered: Hydrodynamical condition of the clogging with silt, design on clogging with silt of weeping hole, reservoirs and city's water pipes filters.)

44. Experimental proof of obtained solution.

Having no possibility in detailing consideration of the experiments which, however, will be done in Chapter IV of this paper (in print), we will make schematic description of experimental models, and the results of the comparison of the theoretical solution with experiments.

*(The experiments were performed under direction of the author by Miss K. T. Gerasimova.)

The experiments were performed on two absolutely different models, according to its designation: On one of them was checked (qualitatively) the model of the movement of the ground stream (which was worked out in this work) saturated with small sandy and clayey particles; on the other was performed the observation of the piezometric pressure, as $f(t, s)$, and of the concentration of the stream as $f(t, s)$, and finally, of the discharge of stream as $f(t)$ only.



Фиг. 11. График функции $H = H(s)$ для различных моментов времени.

Figure 11.

Graphic of the function $H = H(s)$ for different moments of time.

3. The change of pressure as $t(a)$ occurs as were indicated in the
 at the values of the concentration and velocity of the movement of the stream
 and the smaller the diameter of the pores of the ground skeleton.

4. Obtained equation is obviously justifiable only in the portion where

$$\frac{64}{72} \approx 0.89$$

We give the practical interpretation of obtained eq. (11).

Given on Figure 11, curve II, curve II = B(a)
 correspond to the following data:
 $H = 1m, L = 10m, \theta = 20^\circ$
 $N = 0.01, d = 0.002cm$
 and they correspond to the four different
 moments of time:

$$\theta_1 = 2, \theta_2 = 4, \theta_3 = 6, \theta_4 = 8$$

respectively. Therefore in the pres-
 ent paper, all elements of hydrody-
 namical elements of the stream are
 given in final form.

Practical application of obtained
 solution will be given in Chapter
 III, V, VI, and VII, in which the

Figure 11.
 Graphic of the function $B(a)$
 for different moments of time.

Following questions are considered: hydrodynamical condition of the flow-
 ing with slit, design on flowing with slit of weeping holes, reservoirs
 and city's water pipes filters.

4. Experimental proof of obtained solution.

Having no possibility in detail consideration of the experiments
 which, however, will be done in Chapter IV of this paper (in print), we
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The experiments were performed on two absolutely different models,
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 this work) saturated with small sandy and clayey particles; on the other
 was performed the observation of the piezometric pressure, as $t(a)$, and
 of the concentration of the stream as $t(a)$, and finally, of the diameter
 of stream as $t(a)$ only.

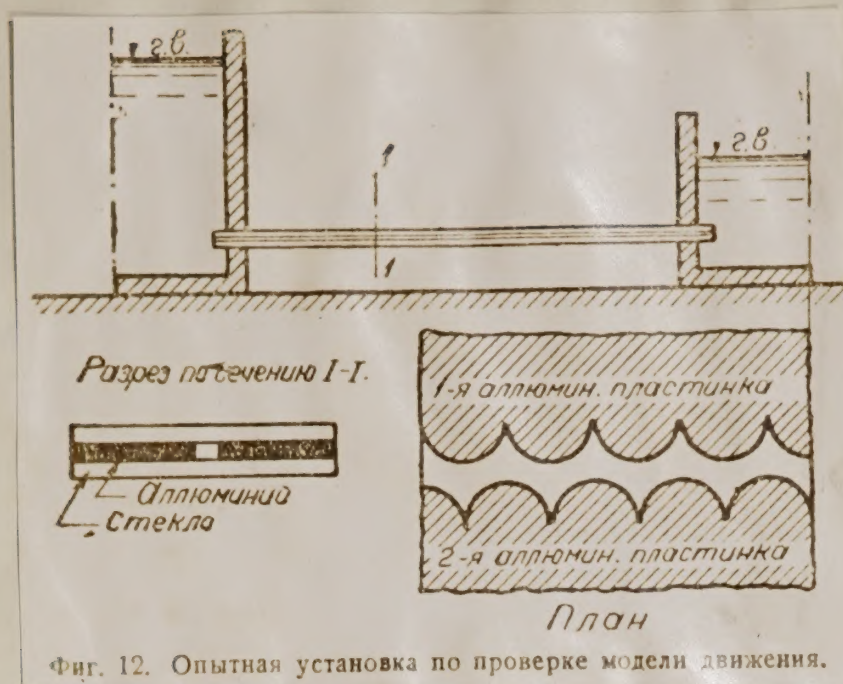


Figure 12.
The experimental set-up for checking of
the model of the movement.

The first set-up consists of (See Fig. 12) two reservoirs (30 cm in height) joined together by the model of the percolation tube which was made out of two plates of alluminum 40 cm. in length, 2 mm. thick and 4 cm. wide. From above and below these plates were covered with glass plates which were fastened to former with special press. The alluminum plates were covered with vaseline for water-proofness. Therefore, in the experiments the only problem that was checked was the one for deposition of silt on the plane.

The technic of the experiments consist of taking photographic pictures in various moment of the time of the picture of the deposition of solid particles and also in immediate observations. The experiments were performed with concentration of stream 1 - 2%; the pressure - 20 - 30 cm. Saturation of the stream was made with particles 0.06 mm. - 0.075 mm., the pores of the percolation tube's model - 0.3 - 0.1 mm. Second set-up (see Fig. 13) consists of covered with glass (from 2 sides) tray (box) of 100 cm. in length, 20 cm. in width and 50 cm in height, in which, with the help of special wooden box-line shield; creates forced movement of the stream. This shield in certain manner attached to the model; for the decrease of filtration along the plane of its attachment with model artificial roughness was made. Along the axis of the shield piezometric tubes were made. Besides, in the bottom of the tray (of the box) were also made control piezometric tubes, which also served as opening for taking out the samples for determination of the stream.

Delivery of the solution into model (box) was accomplished with help of the pump from special reservoir located under model (box). The solution, delivered into the reservoir from clear discharge of the model and also special turbine performed practically satisfactory mixing and checking

Figure 12.
The experimental set-up for checking of
the model of the movement.

The first set-up consists of (see Fig. 12) two reservoirs (30 cm in height) joined together by the model of the percolation tube which was made out of two plates of aluminum 40 cm. in length, 2 mm. thick and 4 cm. wide. From above and below these plates were covered with glass plates which were fastened to former with special brass. The aluminum plates were covered with vasoline for water-proofness. Therefore, in the experiments the only problem that was checked was the one for deposition of silt on the plates.

The technique of the experiments consisted of taking photographic pictures in various moments of the time of the percolation of the solid particles and also in immediate observations. The experiments were performed with concentration of stream 1 - 5%; the pressure - 20 - 30 cm. Saturation of the stream was made with particles 0.06 mm. - 0.075 mm. The pores of the percolation tube's model - 0.2 - 0.1 mm. Second set-up (see Fig. 13) consists of covered with glass (from 2 sides) tray (box) of 100 cm. in length, 20 cm. in width and 30 cm. in height, in which, with the help of special wooden box-line shield, creates forced movement of the stream. This shield in certain manner attached to the model; for the decrease of friction along the plane of its attachment with model artificial roughness was made. Along the axis of the shield piezometric tubes were made. Besides, in the bottom of the tray (of the box) were also made control piezometric tubes, which also served as openings for taking out the samples for determination of the stream.

Delivery of the solution into model (box) was accomplished with help of the pump from special reservoir located under model (box). The solution, delivered into the reservoir from clear discharge of the model and also special turbine performed practically satisfactory mixing and checking

of setting down with time of the solid particles.

Thirty experiments were performed: Concentration was taken as 1 - .25%; saturation was performed with clayey and sandy particles of the size 0.005- cm. The sands of the size 0.3 - 0.05 cm was used for deposition and clogging with silt.

The size of the tray which undergoes of the clogging was such: 10 cm high, 20 cm. wide, 20 - 30 cm long - pressure - 20 - 20 cm.

Figure 13.
Experimental set-up for investigation
of hydrodynamical elements of the
stream, changing in the process of the
clogging. Longitudinal cross-section
of the set-up along the axis.

Turning to the results of the comparison of the theoretical solution with experiments, we note the following.

1. Worked out model of the movement of the ground stream is proved in basic; that is, the solid particles carried by stream, are depositing in those places of percolation tube where the force of inertia arose in moving stream, press them to the walls of the tube.

2. Obtained in present paper, theoretical relation is also proved in basic. The best coincidences are given by functions and somewhat worse . The divergency with the experiments in percentage varies within - 13% to -8%. And finally, the flow of the experimental points lies on the theoretical curve quite satisfactorily (See Chapter IV).

of setting down the time of the solid particles.

These experiments were performed: Concentration was taken at 1 - 2%; saturation was performed with clayey and sandy particles of the size 0.005 - 0.01 mm. The range of the size 0.5 - 0.05 mm was used for saturation and changing with a 1%.

The size of the tray which undergoes of the change was such: 10 cm high, 20 cm wide, 20 - 30 cm long - pressure - 20 - 30 mm.

Figure 13.
Experimental set-up for investigation
of hydrodynamic elements of the
stream, changing in the process of the
clogging. Longitudinal cross-section
of the set-up along the axis.

Turning to the results of the comparison of the theoretical solution
with experiments, we note the following.

1. Worked out model of the movement of the ground stream is proved in practice; that is, the solid particles carried by stream, are depositing in those places of percolation tube where the force of inertia arose in moving stream, press them to the walls of the tube.
2. Obtained in present paper, theoretical relation is also proved in practice. The best coincidences are given by functions and somewhat worse. The discrepancy with the experiments in permeable varies within - 15% to - 5% and finally, the flow of the experimental points lies on the theoretical curve quite satisfactorily (see Chapter IV).